Comparison of OFDM Radar and Chirp Sequence Radar

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Abstract: *The radar waveforms OFDM and chirp sequence are compared in terms of accuracy, resolution capabilities, receiver operating characteristics, required resources and signal processing. We show, that both waveforms can be parametrized in such a way, that they yield the same baseband signal which can be processed using the same detection and estimation algorithms. This key insight reveals, that in this case, the waveforms perform identical. However, OFDM additionally allows simultaneous communication using the radar signal at the cost of increased signal processing.*

1. Introduction

For today's radar applications, especially in the field of advanced driver assistance systems, powerful waveforms are needed. They should not only provide accurate and unambiguous measurements of range and velocity even of weak targets while at the same time eliminating ghost targets, but also they should not be demanding in terms of signal processing in order to be implemented in small sensors with low power consumption and at lowest cost. Two promising waveforms, which are discussed in recent publications are OFDM [1] and chirp sequence [3]. In this work, these two waveforms are reviewed, parametrization issues are discussed and a final comparison is drawn to show their advantages and disadvantages, which can serve as a guideline to choose the proper waveform for a given application.

2. OFDM Radar

OFDM Radar uses an Orthogonal Frequency Division Multiplexing (OFDM) signal known from communications as radar waveform. Algorithmic details are extensively described in [1]. In the following, a short summary of the waveform and its parametrization for radar will be given with the goal of comparing the OFDM waveform with the chirp sequence (CS) waveform.

2.1. Waveform

In OFDM, the frequency band is divided into N subcarriers. The subcarriers are orthogonal to each other, if the subcarrier distance is $\Delta f = \frac{U}{T}$, where $U \in \mathbb{N}$ and T is the OFDM symbol duration. To allow processing in a multipath environment, a cyclic prefix of duration T_{G} is added at the beginning of each OFDM symbol. Furthermore, M OFDM symbols are combined to form an OFDM frame. The OFDM waveform in time-frequency domain is shown in fig. 1a.

The transmitted OFDM signal can be written into a matrix

$$\mathbf{F}_{\mathrm{tx}} = \begin{pmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,M-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,M-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1,0} & a_{N-1,1} & \cdots & a_{N-1,M-1}, \end{pmatrix} \tag{1}$$

where $a_{n,m} \in$ denotes the modulation symbol on subcarrier n of the m-th OFDM symbol. The time domain samples at a sampling rate of $f_{\mathrm{S,OFDM}} = \frac{1}{N\Delta f}$ can be calculated efficiently by applying an inverse fast fourier transformation (IFFT) to each column of $\mathbf{F}_{\mathrm{tx}}$. At the receiver a fast fourier transform (FFT) inverses this process [1].

Assuming a Doppler shift of f_{D}, a two-way time delay of τ and a center frequency f_{c}, then according to [1], the elements of the matrix of the received OFDM echo signal after removal of cyclic prefix can be written as

$$(\mathbf{F}_{\mathrm{rx}})_{n,m} = b(\mathbf{F}_{\mathrm{tx}})_{n,m} \exp\{j2\pi f_{\mathrm{D}} T_{\mathrm{O}} m\} \exp\{-j2\pi(n\Delta f + f_{\mathrm{c}})\tau\} \exp\{j\tilde{\varphi}\}, \tag{2}$$

where b is the attenuation, which can be calculated using the radar range equation [2] and $\tilde{\varphi}$ is an unknown phase shift. Since the transmitted matrix is known, elementwise division of (2) by (1) eliminates the dependency on the modulation symbols:

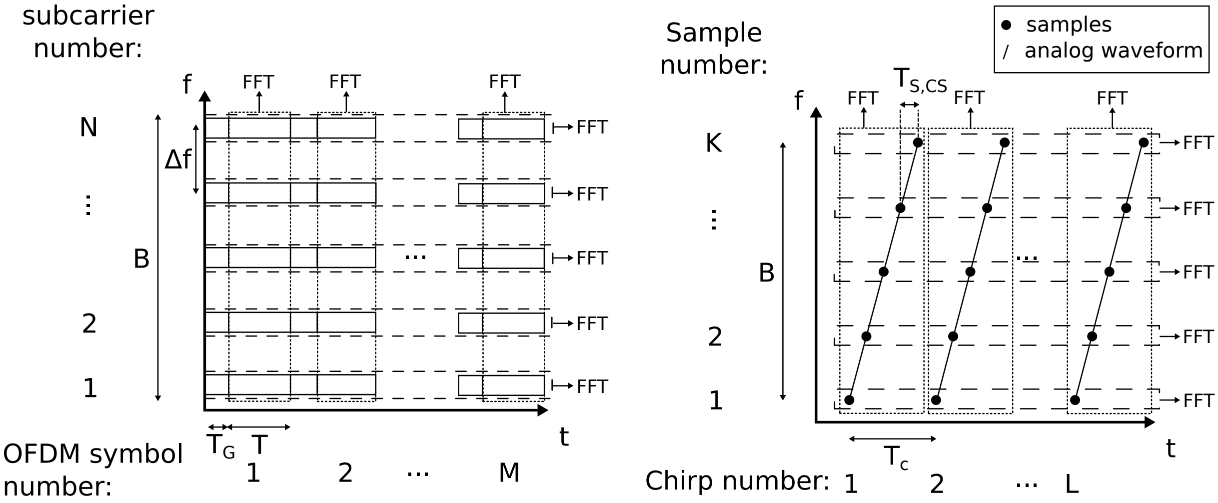
$$(\mathbf{F})_{n,m} = \frac{(\mathbf{F}_{\mathrm{rx}})_{n,m}}{(\mathbf{F}_{\mathrm{tx}})_{n,m}} = b \exp\{j2\pi f_{\mathrm{D}} T_{\mathrm{O}} m\} \exp\{-j2\pi(n\Delta f)\tau\} \exp\{-j2\pi f_{\mathrm{c}}\tau\} \exp\{j\tilde{\varphi}\}. \tag{3}$$

Eq. (3) represents the baseband matrix, which is used for radar processing by means of a two-dimensional FFT as illustrated in fig. 1a. This allows estimating Doppler and range dependant frequency, which can be linearly transformed into the target parameters range and radial velocity. Further details can be found in [1].

2.2. Parametrization

The starting point for radar waveform design are requirements concerning resolution in range ΔR and velocity Δv as well as maximum (unambiguous) range R_{max} and velocity v_{max}. All waveform parameters can be derived from these requirements [1]:

1. Bandwidth: $B = N\Delta f \geq \frac{c}{2\Delta R}$,
2. Number of subcarriers: $N \geq \frac{R_{\mathrm{max}}}{\Delta R}$ and to avoid deorthogonalization: $N \ll \frac{cB}{2v_{\mathrm{max}}}$,
3. Subcarrier spacing: $\Delta f = \frac{B}{N}$,
4. Guard Intervall: $T_{\mathrm{G}} \geq \frac{2R_{\mathrm{max}}}{c}$,



(a) OFDM waveform (b) Chirp sequence waveform

Figure 1: Both waveforms in the time-frequency plane and radar signal processing scheme using two-dimensional FFT.

5. OFDM symbol time: $T = \frac{U}{\Delta f}$, where $U \in \mathbb{N} \Rightarrow T_\mathrm{O} = T + T_\mathrm{G}$, with $T_\mathrm{O} \leq \frac{c}{2 f_\mathrm{c} v_\mathrm{max}}$ $\Rightarrow U_\mathrm{max} = \left\lfloor \left(\frac{c}{2 f_\mathrm{c} v_\mathrm{max}} - T_\mathrm{G} \right) \right\rfloor$,

6. Number of OFDM symbols per frame: $M \geq \frac{c}{2 f_\mathrm{c} \Delta v T_\mathrm{O}}$ with $M T_\mathrm{O} \leq \frac{\Delta R}{2 v_\mathrm{max}}$ to avoid range cell migration,

7. Sampling rate: $f_\mathrm{S,OFDM} = N \Delta f = B$, if common OFDM processing using IFFT and FFT is to be used. Alternative processing (described below) allows a lower $f_\mathrm{S,OFDM} = \frac{N \Delta f}{U}$.

These parameters fully describe the OFDM waveform, which meets the given requirements at lowest possible complexity if bounds are chosen tight. If the sampling rate is chosen according to $f_\mathrm{S,OFDM} = \frac{N \Delta f}{U}$ with $U > 1$, then the time domain samples of the m-th OFDM symbol can be expressed as

$$s_m \left(\frac{i}{f_\mathrm{S}} \right) = \sum_{i=0}^{N-1} a_{n,m} \left(\exp \left\{ j 2 \pi \frac{in}{N} \right\} \right)^U , \tag{4}$$

which differs from an IFFT. If IFFT/FFT processing is to be used for OFDM modulation in the case of $U > 1$, then the high sampling rate $f_\mathrm{S} = N \Delta f = B$ is necessary along with up- and downsampling in freqency domain at the transmitter and receiver, respectively.

Summarized, the resolution requirements determine necessary spectral resources and illumination time, whereas maximum range and velocity requirements determine the size of the two-dimensional FFT of the radar detector and thus the signal processing cost.

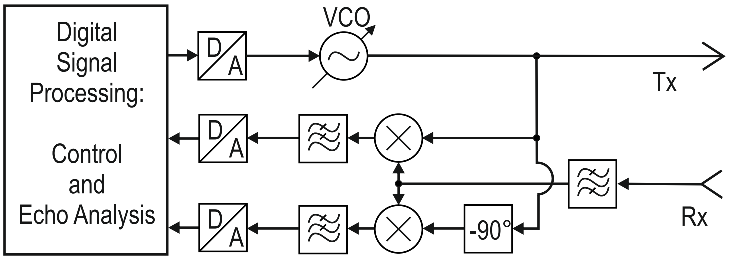


Figure 2: FMCW radar transceiver architecture using a direct conversion quadrature mixer.

3. Chirp Sequence Radar

The CS waveform is a poweful FM radar waveform, which is able to resolve targets unambiguously in range and Doppler [3]. The modulation signal of chirp sequence radar consists of L consecutive linear frequency ramps (chirps) with bandwith B and rise time f_c as shown in fig. 1b. It is processed using a typical FMCW transceiver architecture as shown in fig. 2, where the transmitted signal is directly mixed into complex baseband.

3.1. Waveform

Assuming a target at a distance R_0 with a constant radial velocity of v, the phase of the baseband signal of the l-th ramp can be approximated by [3]

$$\phi_x(k,l) \approx 2\pi\Bigg\{ \underbrace{-\frac{2f_\mathrm{c}v}{c}}_{f_\mathrm{D}} T_\mathrm{c}l + \Bigg(\underbrace{-\frac{2f_\mathrm{c}v}{c}}_{f_\mathrm{D}} \underbrace{-\frac{2BR_0}{c(T_\mathrm{c}-T_\mathrm{ol})}}_{f_\tau} \Bigg) kT_\mathrm{S} - \frac{2f_\mathrm{c}R_0}{c} \Bigg\} \qquad (5)$$

where f_c is the center frequency, c the speed of light, f_D the Doppler frequency and f_τ the Range dependant frequency shift. The approximation holds for $B \ll f_\mathrm{c}$ [3]. Compared to [3], a slight modification has been introduced to account for the overlap time T_ol, which is the time interval at the beginning of a chirp, during which the last part of the previous chirp is still recieved. The resulting beat frequency during this time is of no use and thus should be discarded. This leads to slightly steeper ramps and a bandwidth increase of $T_\mathrm{ol}B/(T_\mathrm{c}-T_\mathrm{ol})$.

$\phi_x(k,l)$ in (5) posesses a two-dimensional structure. The first term, which depends solely on the Doppler frequency f_D is indexed by l and the term depending on the beat frequency $f_\mathrm{B} = f_\mathrm{D}+f_\tau$ is indexed by k. Thus, the complex baseband signal with amplitude a and $\tau = \frac{2R_0}{c}$ may be written in matrix notation, yielding [3]

$$(\mathbf{M})_{k,l} = a\exp\{j\phi_x(k,l)\} = a\exp\{j2\pi f_\mathrm{D}T_\mathrm{c}l\}\exp\{j2\pi f_\mathrm{B}T_\mathrm{S,CS}k\}\exp\{-j2\pi f_\mathrm{c}\tau\}\,. \qquad (6)$$

The two frequencies f_D and f_B which manifest along rows and columns, respectively, can be estimated using a two-dimensional FFT as hinted in fig. 1b and subsequently may be transformed into the target parameters R and v.

3.2. Parametrization

As in OFDM Radar, the starting point for deriving design parameters of the CS waveform are requirements concerning ΔR, Δv, R_{max} and v_{max}:

1. Bandwidth: $B \geq \frac{c}{2\Delta R}$, as in OFDM,
2. Number of effective samples per chirp: $K \geq \frac{R_{\text{max}}}{\Delta R}$, as N in OFDM,
3. Maximum overlap time: $T_{\text{ol}} = \frac{2R_{\text{max}}}{c} = T_{\text{G}}$,
4. Chirp duration: $T_{\text{c}} \leq \frac{c}{2f_{\text{c}}v_{\text{max}}}$, as T_{O} in OFDM,
5. Number chirps per sequence: $L = \frac{c}{2f_{\text{c}}\Delta v T_{\text{c}}}$, with $LT_{\text{c}} \leq \frac{\Delta R}{2v_{\text{max}}}$ to avoid range cell migration, as M in OFDM,
6. Sampling rate: $f_{\text{S,CS}} = \frac{1}{T_{\text{S,CS}}} \geq \frac{T_{\text{c}}}{K}$,

where choosing tight bounds leads to effective use of resources.

4. Comparison

Both waveforms achieve true two-dimensional resolution (in both range and Doppler). Furthermore, they can be parametrized in such a way, that the information carrying signals of both waveforms are identical. A precondition for that is $\frac{B}{T_{\text{c}}} \gg 1$ (very steep ramps of the CS waveform), because then, $f_{\text{B}} \approx f_{\tau}$ in (6) and thus

$$(\mathbf{M})_{k,l} \approx a \exp\{j2\pi f_{\text{D}} T_{\text{c}} l\} \exp\{-j2\pi k(B/(T_{\text{c}} - T_{\text{ol}})T_{\text{S,CS}}\tau\} \exp\{-j2\pi f_{\text{c}}\tau\}. \quad (7)$$

A comparison with (3) shows that both signals are identical except for the phase term $\tilde{\varphi}$, which is a nuissance parameter for the estimation problem [1] and thus can be neglected, if

- $T_{\text{c}} = T_{\text{O}}$,
- $T_{\text{S,CS}} = \frac{T_{\text{c}}-T_{\text{ol}}}{B}\Delta f = \frac{T_{\text{c}}-T_{\text{ol}}}{BT} = \frac{T_{\text{c}}-T_{\text{ol}}}{B(T_{\text{O}}-T_{\text{G}})} = \frac{1}{B} = T_{\text{S,OFDM}}$, if $T_{\text{G}} = T_{\text{ol}}$
- $a = b$

Therefore, in this case, both waveforms posses identical properties in terms of accuracy, resolution, receiver operating characteristic, update rate and are equally demanding for the radar signal processor. However, OFDM has the extra cost of the OFDM modulator and demodulater but posseses the ability to provide simultaneous communication.

The identified advantages and disadvantages of both waveforms are summarized in tab. 1.

Table 1: Comparison of OFDM and fast chirp radar waveforms

Criteria	OFDM	Chirp Sequence
Analog hardware	standard transceiver architecture, quadrature mixer, mixer for down-conversion	VCO or DDS, quadrature mixer as direct conversion mixer
Preprocessing	OFDM modulation and demodulation, storage of $2NM$ symbols, NM divisions	storage of NM symbols
Radar signal processing	two-dimensional periodogram	
communication	possible	not possible
Advantages	spectrum and hardware efficient method of combined radar and communication	less hardware complexity, no de-orthogonalization issues and no constraints for chirp duration
Recommended usage	applications where both radar and communications are needed	applications without a need for communications

5. Conclusion and Outlook

In this work, we have reviewed the two radar waveforms OFDM and chirp sequence and summarized the important steps to choose their parameters for given application requirements. Furthermore, we have compared both waveforms with respect to performance and required hardware and signal processing complexity. We have found, that the baseband signal of chirp sequence converges to the signal used in OFDM radar for detection (after preprocessing and symbol removal), if the steepness of frequency ramps approaches infinity. Thus, both waveforms will perform identical in that case, which is the key result of our work.

Since convergence is only achieved in the limit, simulations could help to compare the performance in different modes of operation, which is a current research task of the authors.

References

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