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Capacity Regions for Wireless Multi-Channel Ad Hoc Networks

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IEEE 74th Vehicular Technology Conference, San Francisco, 5 - 8 September 2011

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Overview

Capacity Regions of Wireless Multi-Channel Ad Hoc Networks

- **Introduction: Motivation and contribution**
- **System model: Calculating capacity regions**
- **Influence of FDMA on the capacity region**
- **Conclusion: Main points**

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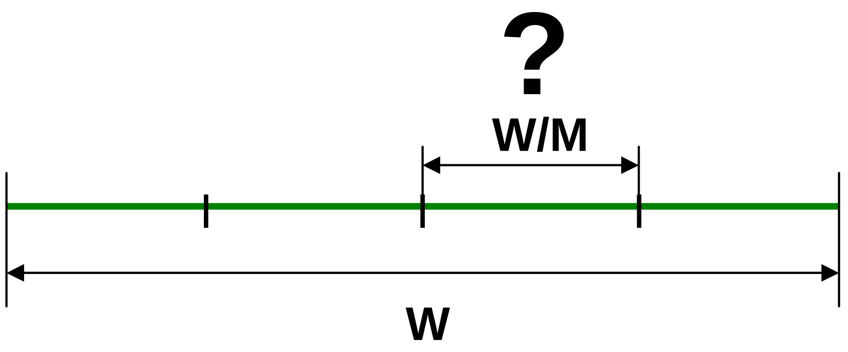
Motivation and contribution

Limited HF bandwidth, a lot of nodes: Need multi-channel networks!



Capacity regions of *ad hoc* networks with multiple channels ?

Modeling of network capacity

- **Topological model according to Toumpis & Goldsmith[1]**
  - (+) deterministic results
  - (+) Impact of a fixed topology can be examined
  - (−) Due to complexity, model is limited to a small number of nodes

Contribution

- **We extend the model of Toumpis & Goldsmith to show the influence of FDMA, i.e., multiple channels on the capacity region**

[1] S. Toumpis, A.J. Goldsmith, *Capacity Regions for Wireless Ad Hoc Networks,* IEEE Transactions on Wireless Communications, Juli 2003.

System model 1/4: Basic definitions

Basic model: Fixed topology, half-duplex, Shannon Capacity as metric

Network with n nodes

- Arbitrary but fixed positions
- Half-duplex directional transmission
- Transmitter A_i adapts data rate to receiver A_j according to Shannon capacity
- Path loss model for transmission
- Constant transmission power

P: Transmission power

d_{ij}: distance to receiver

α: path loss exponent

η: noise power density

W_m: channel bandwidth

T: set of active transmitters

Transmission rate

$$r_{ij} = W_m \log_2 \left(1 + \frac{d_{ij}^{-\alpha} P}{\eta W_m + \sum_{k \in T, k \neq i} d_{kj}^{-\alpha} P}\right)$$

System model 2/4: Network states

Transmission protocol defines possible network states

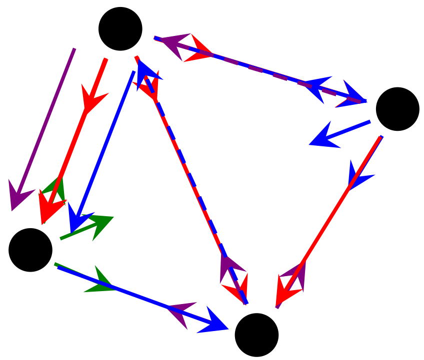
Transmission protocol

- **A transmission protocol has a set of allowed transmissions**

Set of states

$$\{S_i | i = 1, ..., N\}$$

Static multihop routing:



***Rate vector* R(S_i)**

- **Mathematical representation of a state in vectorial form**
  - n (n-1) dimensional (n nodes)
  - Each element is the data rate of a possible transmission

***Time Division Schedule* τ**

- **Linear combination of states, subject to a unit time contraint**

R(τ)

$$R(\tau) = \sum_{i=1}^{k} a_i R(S_i), \quad \sum_{i=1}^{k} a_i = 1$$

System model 3/4: *Capacity Region*

The *Capacity Region* is the set of transmission rates reachable by time division schedules

Basic rate vectors

$$\Psi = \{R(S_i) | i = 1, ..., N\}$$

Convex hull

$$C_0 = \text{conv}\ \Psi$$

Causality contraint

- **Information is routed**
  $\rightarrow$ negative rates
- **Negative elements in linear combinations**
  $\rightarrow$ acausal transmission

➡ V_n **: Set of all vectors without negative elements**

Capacity Region

$$C = C_0 \cap V_n$$
$$= \left\{ \sum_{i=1}^{N} a_i R(S_i) : a_i \geq 0, \sum_{i=1}^{N} a_i = 1 \right\} \cap P_n$$

➡ **n (n-1) dimensional closed subspace**

System model 4/4: *Uniform Capacity*

Characteristic value of the *Capacity Region*: *Uniform Capacity*

Uniform Capacity

- **Multiple of maximum rate, achievable by all transmissions concurrently**

  → Bisection r^{opt} with the edge of the *Capacity Region*

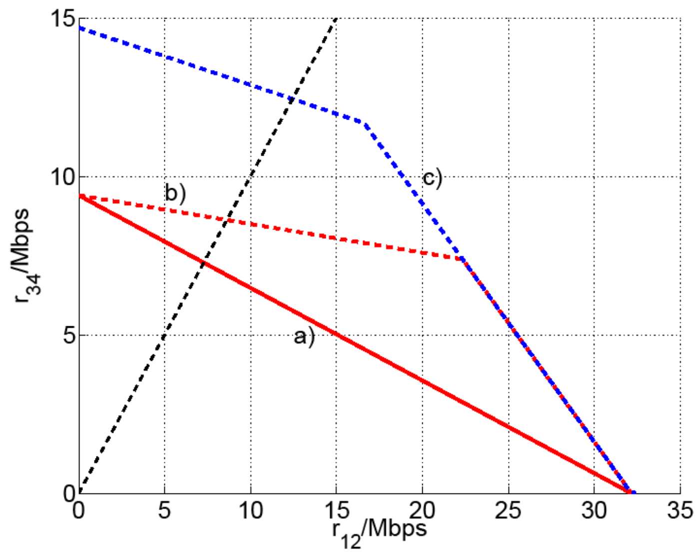
$$C_u = \sum_{i=1}^{n(n-1)} r_i^{\text{opt}}$$

Calculation

- **Iterative search along constant rate vector**
- **Check of point in capacity region by linear programming**

$$\min\{c^\top a | \Psi \cdot a = b, 1 \geq a \geq 0\}$$

Cut through *Capacity Region*



a) SH, no parallel transmissions
b) SH, parallel transmissions
c) MH, parallel transmissions

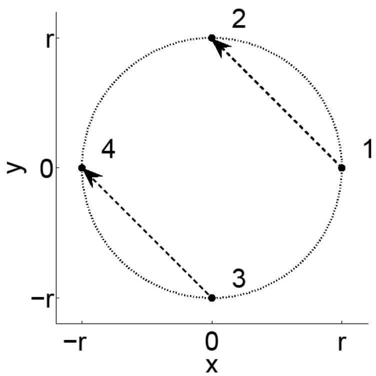
Some observations for single transmission rates

Rates depend on network radius

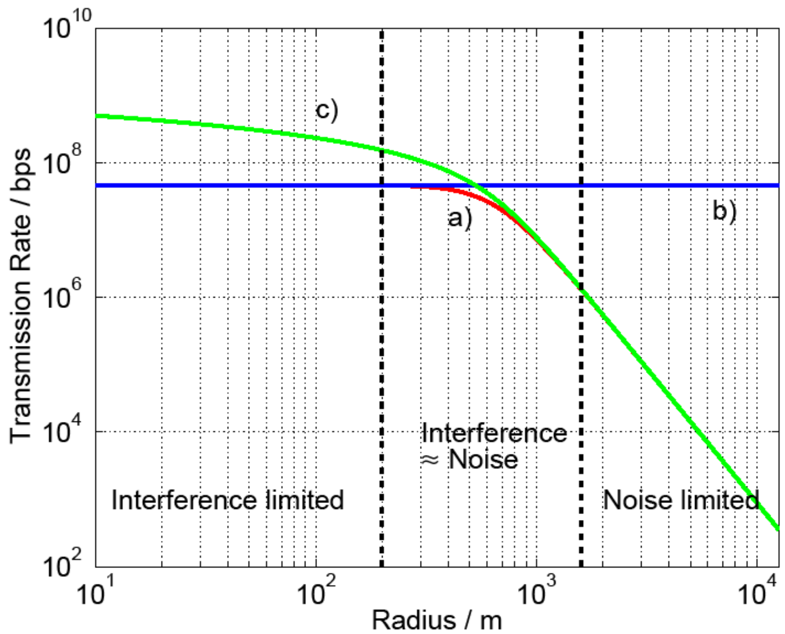
System parameters

- **Constant system parameters**
- **Influence of interference and noise depends on network radius r**

Topology



Transmission rate



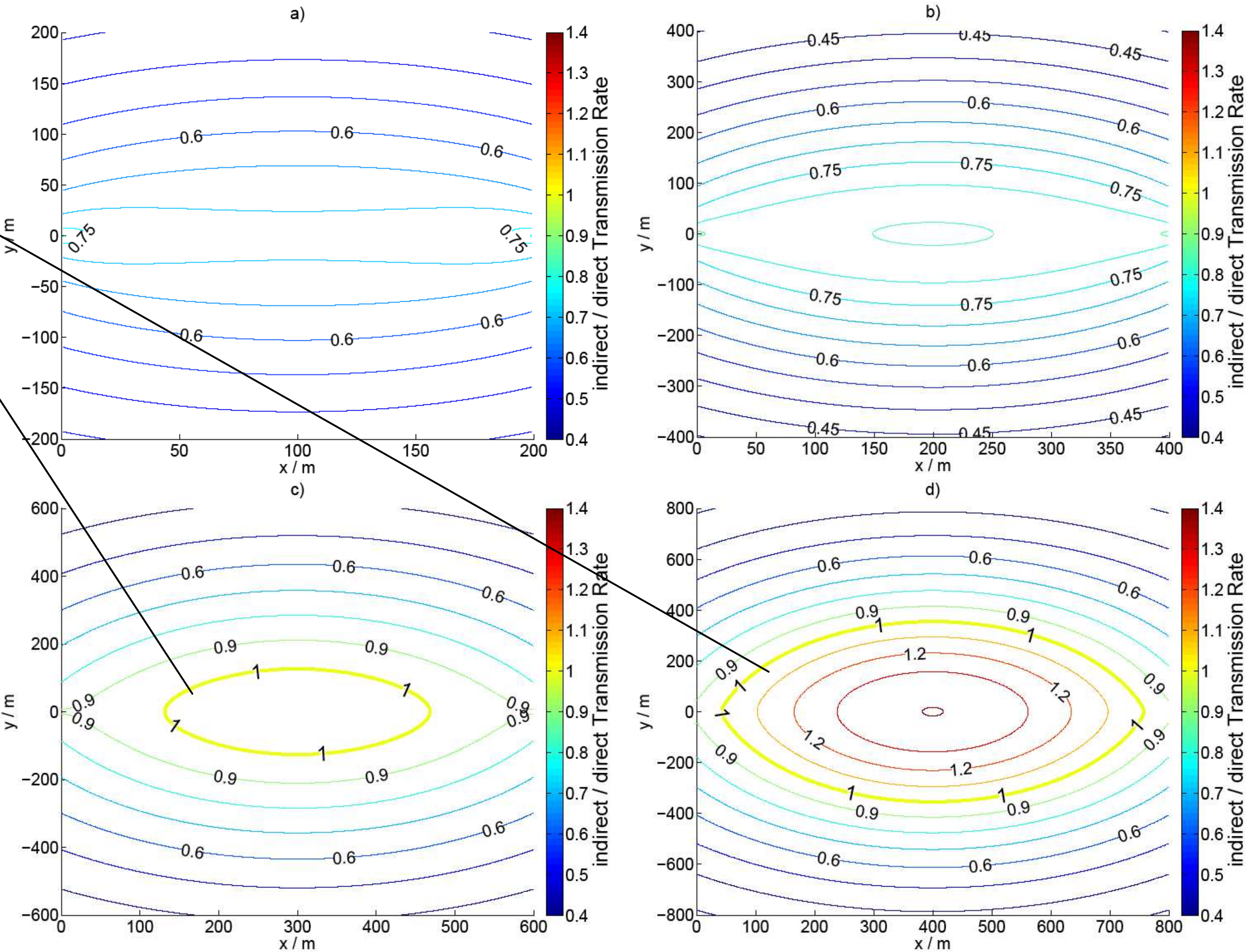
a) Exact rate
b) Noise neglected
c) Interference neglected

P = 0.1 W, α = 4, T = 293, η = kT = 4 e-21 W/Hz
W = 20 MHz

Single rates: *Relay*-Position

Optimal position of *relay node* depends on distance between transmitter and receiver

Ratio > 1



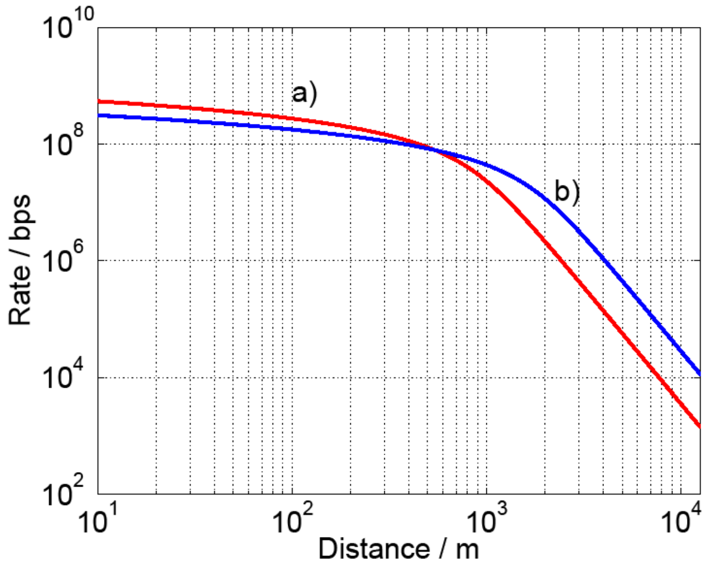
- **Minimum distance needed for rate gain**

Optimal position: bétween receiver and transmitter

Single rates: *Multi Hop Routing*

Improvement by *Multi Hop Routing* depends on distance

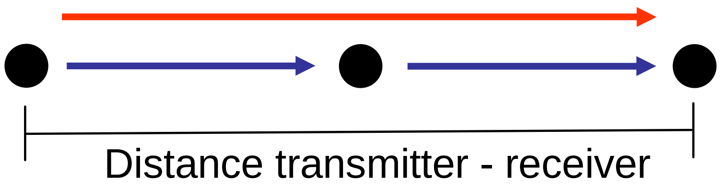
Transmission rate



a) Rate direct transmission
b) Rate indirect transmission, optimal relay position

Multi Hop Routing

- ***Multi Hop Routing*** **beneficial for given scenario only if distance greater than approx. 600 meters (interference neglected)**

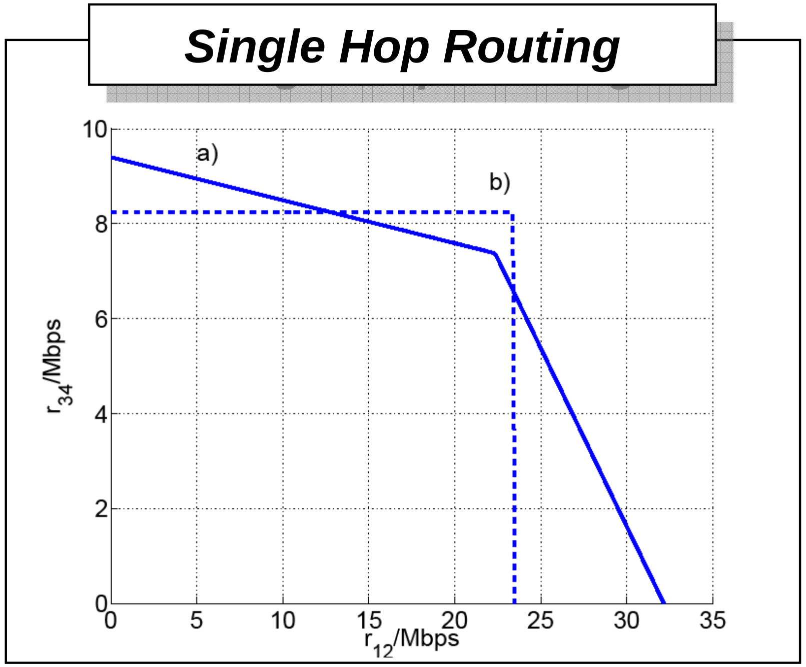


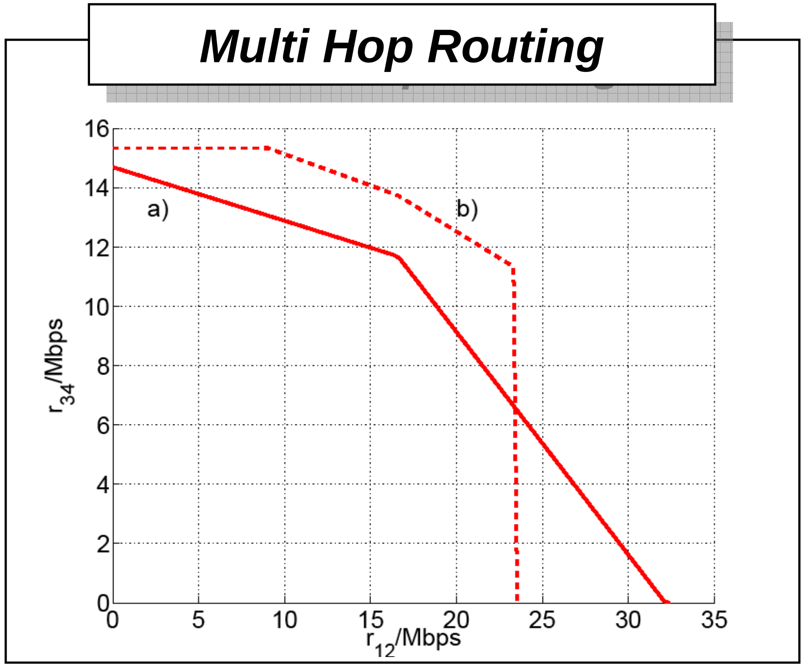
Influence of FDMA

Orthogonalization in frequency possible

Multiple channel case

- **Parallel transmissions are independent, if they use orthogonal channels**
- **Less bandwidth, hence lower maximum transmission rates**





a) Parallel transmission in the same channel with bandwidth W
b) Parallel transmission in separate channels with bandwidth W/2

Channel assignment

New degree of freedom: assignment of transmission channels

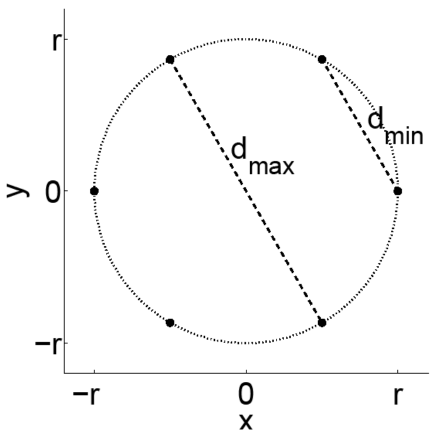
Two possibilities

Case 1	Case 2
▪ Dynamic adaptation for every transmission state	▪ One fixed assignment for all possible states

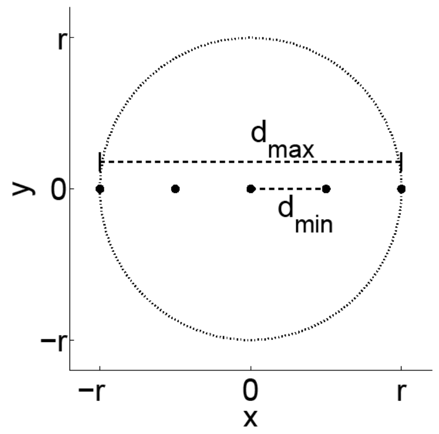
Canonical topologies

Topologies differ in inter-node distances, we consider extreme cases

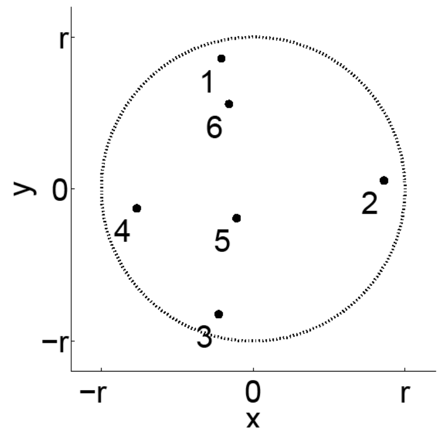
Circular



Linear



Mixed

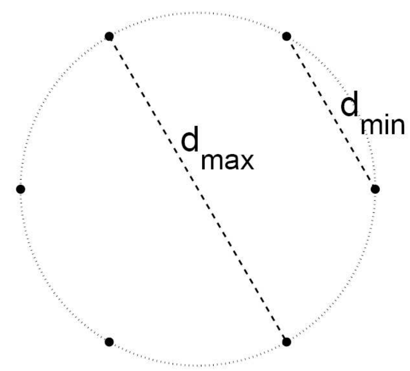


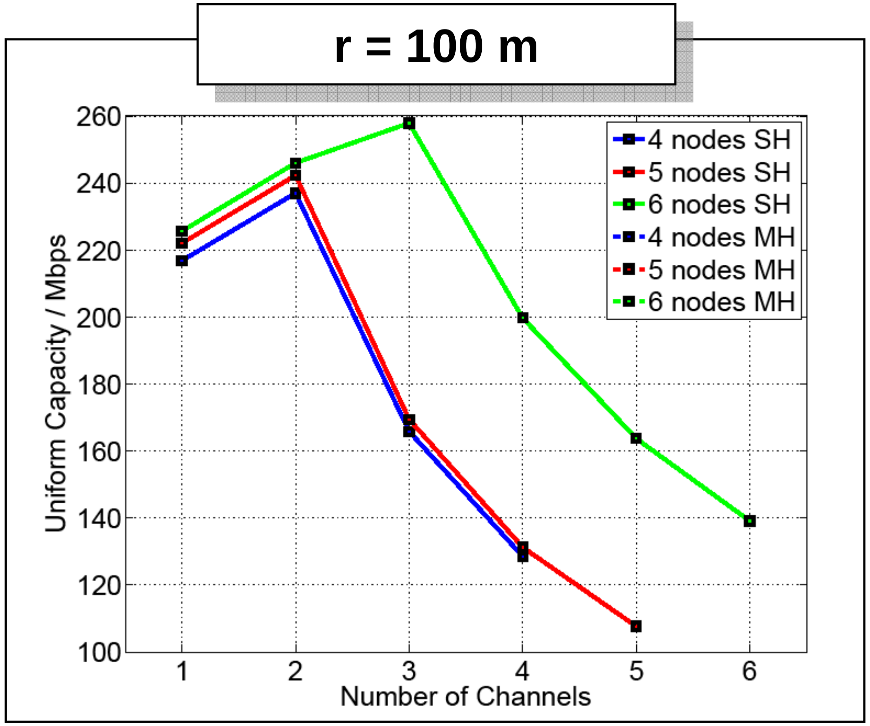
- **Strong influence on capacity region**
- **Canonical topologies:**
  - Extreme cases
  - Distance depend linearly on radius
  - For small n: Difference of d_{min} and d_{max} is significantly less in circular networks, compared to linear networks

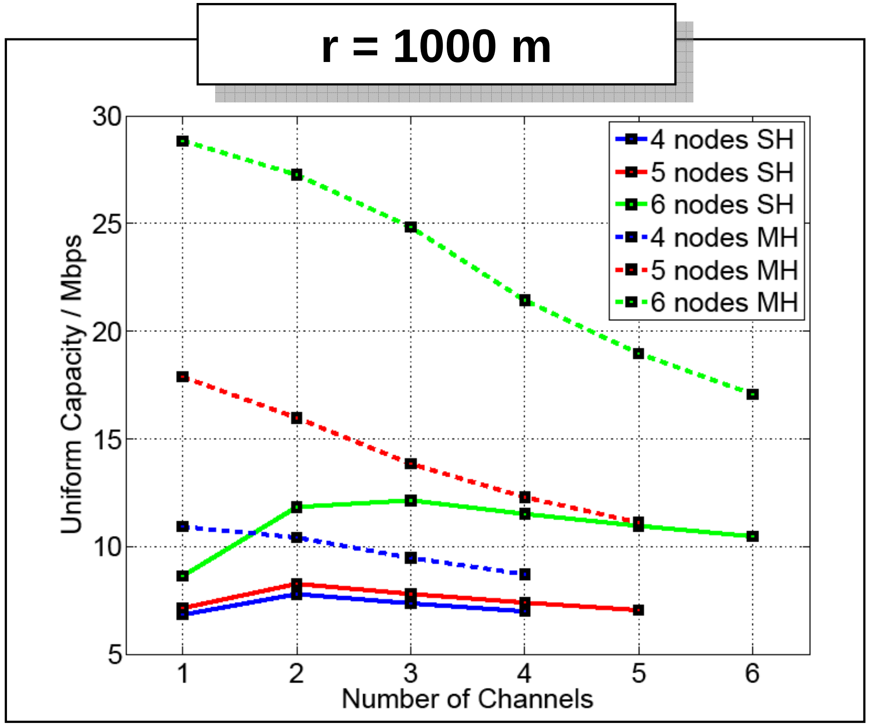
Case 1

Circular topology

- **Small network size:**
  - Interference avoidance through FDMA increases capacity
- **Large network size:**
  - SH: Higher relative increase
  - MH: Multi-hop routing already reduces interference, no gain, *Spatial Reuse* better



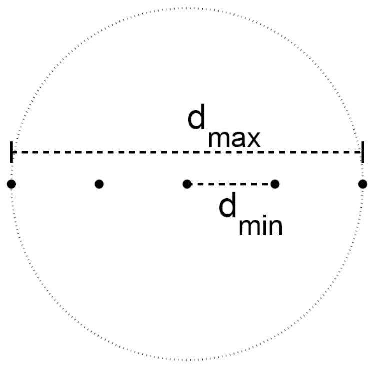


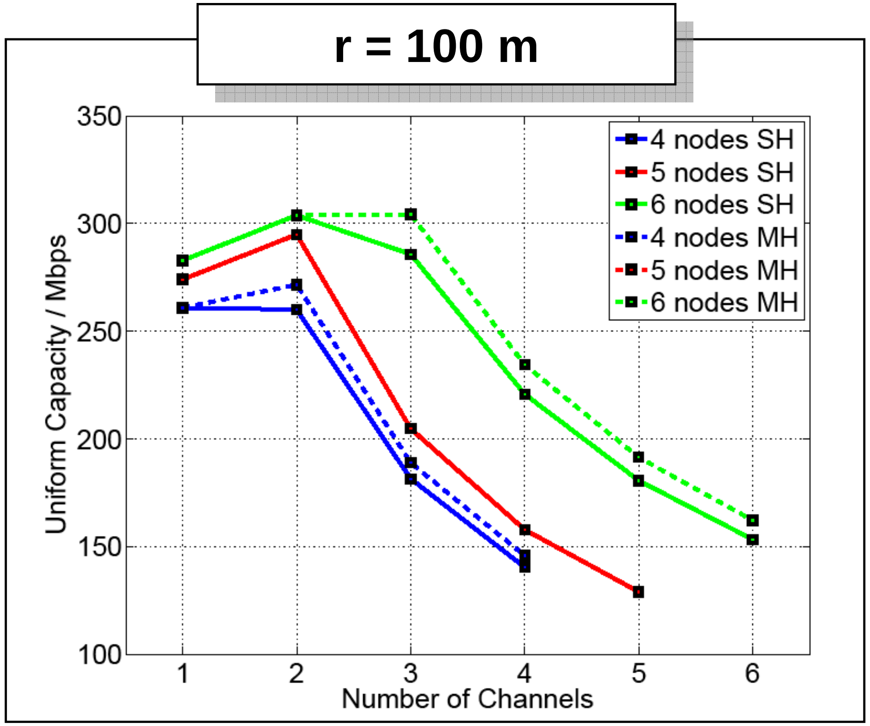


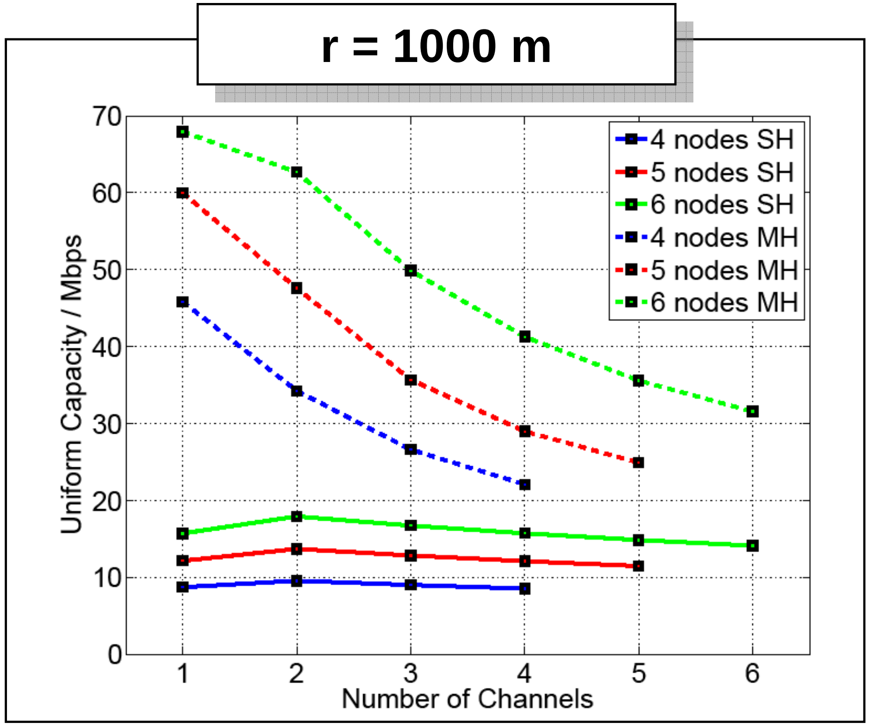
Case 1

Linear topology

- In principle same results
- Difference through greater variation between d_{min} and d_{max}
- Interference avoidance of *Multi Hop Routing* visible at small radii, since distances to interferers are smaller



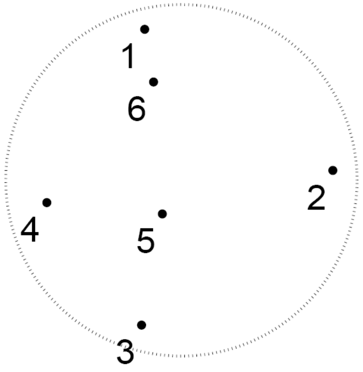




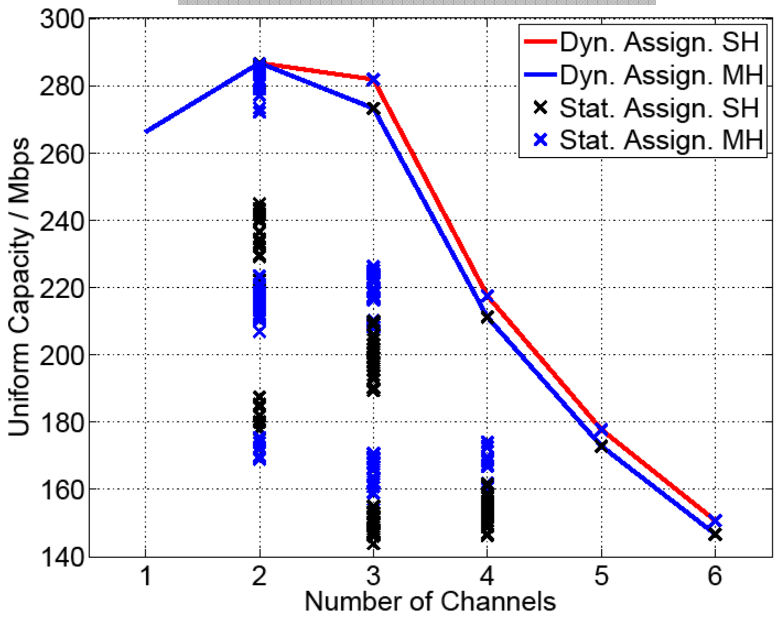
Case 2

Mixed topology

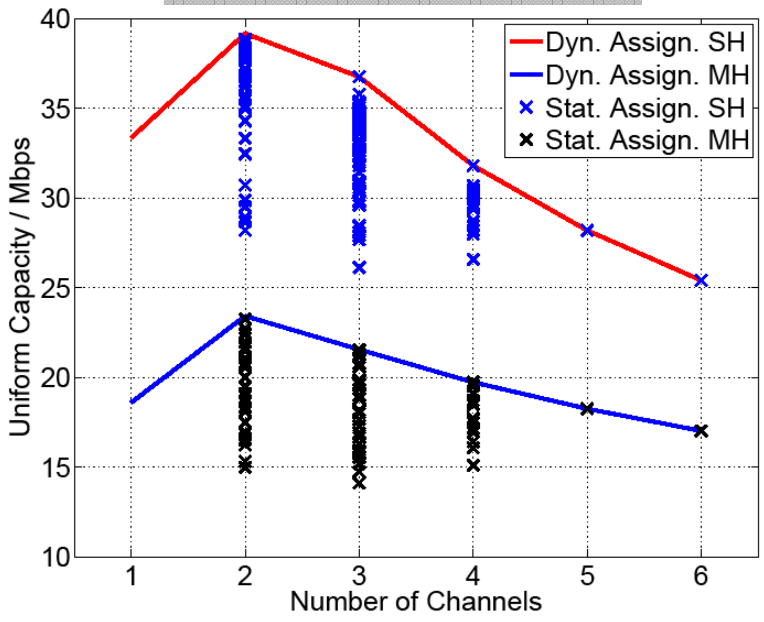
- High variation between good and bad assignments
- Optimal fixed channel assignment almost as good as dynamic assignment



r = 100 m



r = 1000 m



Conclusion: Main points

FDMA increases capacity, if self-interference is dominant

1. **Influence of FDMA on the capacity of ad hoc networks**
   - Increases capacity, if self-interference is dominant
   - Decreases capacity, if noise is dominant

2. ***Multi Hop Routing* decreases influence of self-interference**

   ⇒ FDMA is less beneficial

3. **In networks with great variance of inter-node distances, both effects play a role**

   ⇒ Good strategy: Orthogonalization of nearby interferers, multi-hop transmission to far away receivers

4. **Simulation results indicate that fixed channel assignment for a given topology does not affect the capacity region.**

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Discussion / Q&A

Limits of the system model

Model is limited to networks with a small number of nodes

Number of states
▪ **Combinatorical modelling lets number of states explode** ▪ Need to calculate rate vectors ▪ LP becomes more complex ▪ **Complexity LP: polynomial[1]** ▪ **Number of states grows factorially**

n	N
3	13
4	145
5	1041
6	19.651
7	196.813
8	5.227.713
40	≈ $3 * 10^{61}$

Number channel assignments
▪ **Exponential growth (M^n)** ▪ **Can remove redundant assignments**

n	M	M^n	Reduziert
3	2	8	3
4	2	16	7
4	3	81	6
5	2	32	15
5	3	243	25
5	4	1024	10
5	5	3125	1

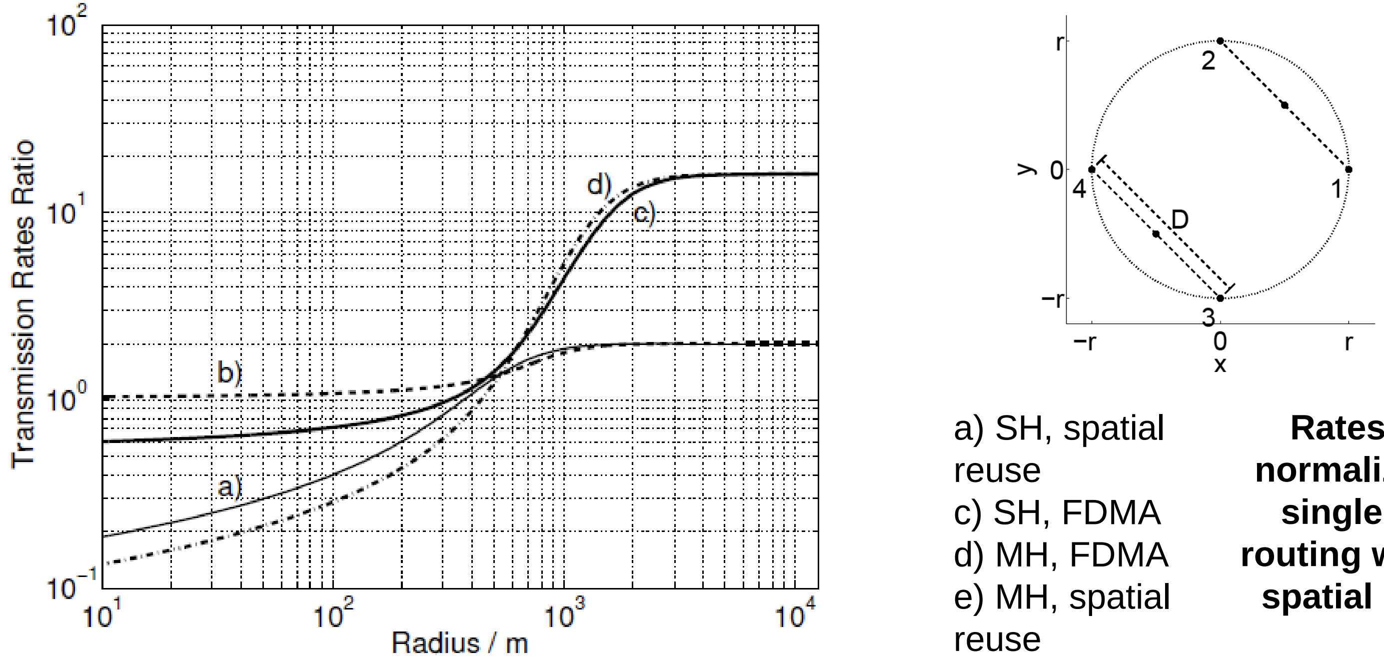
[1] F. Jarre, J. Stoer, *Optimierung,* Springer-Lehrbuch. Springer, Berlin, 2004.

Single rates: TDMA, spatial reuse, FDMA

Best choice depends on network radius

Different transmission protocols

1. **TDMA:** Orthogonalization in time
2. ***Spatial Reuse*:** Transmission in same channel
3. **FDMA:** Transmission in separate channels with bandwidth W/2



a) SH, spatial reuse
c) SH, FDMA
d) MH, FDMA
e) MH, spatial reuse

Rates are normalized to single hop routing without spatial reuse