Narrow- and broadband Interference Robustness for OOK/BPPM based Energy Detection

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Abstract—An analysis of narrow- and broadband interference robustness within an On-Off Keying/Binary Pulse Position Modulation based noncoherent multiband impulse radio ultrawideband communication system is presented. Using the energy detector’s processing gain closed-form expressions of the noise and interference related second order moment statistics at the output of an energy detection receiver are derived. This allows separate statements on the relative modulation specific processing gain with respect to various interference parameters.

Index Terms—Energy detection, interference robustness, processing gain, UWB, OOK, BPPM.

I. INTRODUCTION

The fundamental resolutions of e.g. the Federal Communications Commission (FCC) [1] in 2002 or the Electronic Communications Committee (ECC) [2] in 2008 have lead to enormous research activities in academia and industry to enable a possible commercialization of the unlicensed ultrawideband (UWB) technology. Thereby, wireless community considers UWB as a possible candidate to revolutionize high-speed data transmissions as well as an enabler for the personal area networking industry aiming at novel innovations and a greater quality of the services to the end user.

As future UWB systems are required to be realized with low-complexity and in a power efficient way, a notable part of research focuses recently on suboptimal noncoherent UWB systems [3]. In contrast to coherent UWB systems, the main reasons to favor noncoherent UWB systems are

- the avoidance of a receiver-side high cost, high-speed and power consuming analog-to-digital converter,
- the relaxed synchronization constraints and
- the efficient handling of energy capture resulting from the high multipath diversity.

A promising noncoherent UWB system suited for high data rate transmissions is multiband impulse radio UWB (MIR UWB) [4], [5], [6], [7]. Therein, extremely short pulses are emitted in the same bandpass filterbank covering the available spectrum. The pulses at the filters’ outputs are modulated with On-Off Keying (OOK)/Binary Pulse Position Modulation (BPPM), added up and afterwards transmitted. The receiver front-end uses the same bandpass filterbank to split up the received UWB signal followed by a parallelized energy detection.

A pivotal vulnerability of the energy detection receiver is its high sensitivity with respect to interference. Interference passing the filterbank might lead to a reduction of the instantaneous signal-to-interference-plus-noise ratio (SINR) and hence to a falsified decision process. For this reason, it is required to investigate the interference robustness of an energy detector regarding, e.g., different modulation schemes.

This paper bases on [8] which analyzes out-of-band interference for noncoherent UWB systems with BPPM based energy detection. Recently, [9] extends this approach to make investigations of in-band narrowband interference (NBI) for OOK/BPPM based energy detection. However, a general analytical investigation of narrow-and broadband interference robustness for energy detection of OOK/BPPM is not yet done to the best of the authors’ knowledge.

The remainder of the paper is as follows: In Section II the signal model for OOK/BPPM based data transmission including the energy detection under interference is introduced. Section III defines the processing gain (PG) of the energy detector and analyzes the interference and noise related statistics at its output. The subsequent Section IV identifies the interference robustness for both modulation schemes using the energy detector’s PG. Concluding remarks are given in Section V.

II. SIGNAL MODEL

A. Transmitter

A binary data transmission link within an MIR UWB subband of bandwidth $B$ is considered. Based on OOK/BPPM modulation the rectangular pulse

$$ p(t) = \begin{cases} \frac{2}{T} \cos(2\pi f_c t) & 0 < t < T_p, \\ 0 & \text{else,} \end{cases} $$

(1)

with carrier frequency $f_c$ and pulse duration $T_p$ is emitted with energy $1 (f_c \gg 1/T_p)$. The resulting signal to be transmitted conducts to

$$ s_O(t) = \sqrt{E_p^O} \sum_{k=-\infty}^{\infty} b_k p(t-kT_b), $$

(2)

for OOK and

$$ s_P(t) = \sqrt{E_p^P} \sum_{k=-\infty}^{\infty} p(t-kT_b - b_k T_b) $$

(3)

for BPPM. Data bit $b_k \in \{0,1\}$, which is assumed to be uniformly distributed, is specified by bit energy $E_b$ as well as bit duration $T_b = \frac{T}{d_b}$ with duty cycle $d_b \leq \frac{1}{2}$. Finally,
of mean $E_b$ and second order moment $2E_b^2$. The component
$\Delta x^O = x_{sjn}^O + x_{jn}^O$ contains the mixed signal-noise and signal-interference term

$$x_{sjn}^O = \begin{cases} 0 & b_k = 0, \\ 2\sqrt{\frac{2E_b}{T_p}} \int_0^{T_p} \cos (2\pi f_c t) (J (t) + N (t)) \, dt & b_k = 1, \end{cases}$$

(10)
as well as the contribution

$$x_{jn}^O = \int_0^{T_p} (J (t) + N (t))^2 \, dt \quad b_k = 0, 1,$$

(11)
due to noise and interference-only.

**BPPM:** In contrast to OOK the BPPM decision variable at the output of energy detection is symmetric [8]:

$$x^P = \int_0^{T_p} y^2 (t) \, dt - \int_\frac{T_p}{4}^{\frac{3T_p}{4}} y^2 (t) \, dt$$

$$= x^P + \Delta x^P.$$

(12)
The decision variable $x^P$ compares energy values within two observation intervals of duration $T_p$. It is composed of a signal-only contribution

$$x_s^P = \begin{cases} E_b & b_k = 0, \\ -E_b & b_k = 1, \end{cases}$$

(13)
which is characterized by mean zero and second order moment $E_b^2$. The additional term $\Delta x^P = x_{sjn}^P + x_{jn}^P$ is composed of a mixed signal-noise and signal-interference component

$$x_{sjn}^P = \begin{cases} \frac{a}{2} \int_0^{T_p} \cos (2\pi f_c t) (J (t) + N (t)) \, dt & b_k = 0, \\ -a \int_\frac{T_p}{4}^{\frac{3T_p}{4}} \cos (2\pi f_c (t - \frac{T_p}{2})) (J (t) + N (t)) \, dt, & b_k = 1, \end{cases}$$

(14)
with $a = 2\sqrt{2E_p^2/T_p}$ and the noise and interference-only part

$$x_{jn}^P = x_{jn}^O - \int_\frac{T_p}{4}^{\frac{3T_p}{4}} (J (t) + N (t))^2 \, dt.$$

(15)

### III. Analysis of Interference Robustness

To make statements on the interference robustness of energy detection for OOK/BPPM a proper quality criterion has to be introduced. A possible measure is the PG of an energy detector. It refers the available SINR at the energy detector’s output to the SINR at its input. For OOK this can be described as

$$\text{PG}^O = 10 \log_{10} \left( \frac{2E_b^2}{2Q_1^2 + Q_2^2} \right) - 10 \log_{10} (\text{SINR}_{\text{in}}),$$

(16)
which differs from the PG of the BPPM based energy detection receiver expressed as

$$PG^p = 10 \log_{10} \left( \frac{E_p^2}{Q_p^2 + Q_p^2} \right) - 10 \log_{10}(\text{SINR}_m). \quad (17)$$

In (16) and (17) $Q^p_i$, $i \in \{O, P\}$ stands for the second order moment of the mixed signal-noise and signal-interference component $x_{sjn}^i$, $i \in \{O, P\}$. In contrast, $Q^p_i$, $i \in \{O, P\}$ describes the second order moment of the noise and interference-only part $x_{jn}^i$, $i \in \{O, P\}$.

Based on PG, separate statements on the performance detection can be made for each modulation scheme, i.e., a low modulation specific PG indicates an increased error probability and vice versa. Hence, the smaller $Q^O_i$ and $Q^P_i$, $i \in \{O, P\}$ the lower the modulation related error detection probability. In the following, $Q^O_1$ and $Q^P_2$, $i \in \{O, P\}$ are determined for both modulation schemes.

**OOK:** For OOK, the second order moment of the signal-noise and signal-interference part $x_{sjn}^O$ can be formulated as $(\tau = t_1 - t_2)$

$$Q^O_1 = \frac{8E_p^O}{T_p} \int_0^{T_p} \int_0^{T_p} \cos(2\pi f_c t_1) \cos(2\pi f_c t_2) \cdot [E(J(t_1)J(t_2)) + E(N(t_1)N(t_2))] dt_1 dt_2$$

$$= \frac{4E_p^O}{T_p} \int_0^{T_p} \int_0^{T_p} (R_3(\tau) + R_N(\tau)) \cdot \cos(2\pi f_c (t_1 - t_2))$$

$$+ \cos(2\pi f_c (t_1 + t_2))] dt_1 dt_2. \quad (18)$$

A solution of $Q^O_1$ can be found using Parseval’s theorem under the assumptions $2|f_c + f_1| \gg B_1$ and $4f_c \gg B$. This leads to the closed-form expression

$$Q^O_1 = \sum_{n=0}^{\infty} (-1)^n \left( \frac{2\pi f_c}{2n+1} \right)^{2n} \left( \frac{(2n+1)!}{2n+1} \right) \frac{u_{n,l}}{2\pi f_c (2n+1)}$$

$$- \frac{2\pi f_c}{2n+2} \sum_{n=0}^{2n+1} \frac{u_{n,l}}{2n+2} + \frac{4\pi f_c}{2n+1} \left( \frac{2n+1}{2n+1} \right)$$

$$+ \sum_{n=0}^{\infty} \left( \frac{(2\pi f_c T_p)^{2n+1}}{2n+1} \right) \frac{w_{n,l} + \sin(4\pi f_c T_p)}{2n+1} \sum_{l=0}^{2n+1} z_{n,l},$$

whereas, with $\Delta_{f,k} = f_c - f_k$, the following notations are used:

$$r_{\nu} = \frac{1}{B_1} \left( \frac{(B_1}{2} + \Delta_{f,k})^\nu - \left( \frac{B_1}{2} + \Delta_{f,k} \right)^\nu \right),$$

$$w_{n,l} = \frac{\sin(4\pi f_c T_p + \frac{\nu}{2})}{(2n+1)! (4\pi f_c T_p)^{2n+1}},$$

$$u_{n,l} = w_{n,l} + \frac{(-1)^l}{(2n+1)!} \sum_{k=0}^{l} \sin(4\pi f_c + \frac{\nu}{2} k \pi) \cdot (l - k)!(4\pi f_c T_p)^{k},$$

$$z_{n,l} = \frac{\cos(4\pi f_c + \frac{\nu}{2} k \pi)}{(2n+2)!} \sum_{k=0}^{l} \cos(4\pi f_c + \frac{\nu}{2} k \pi) \cdot (l - k)!(4\pi f_c T_p)^{k},$$

Eq. (19) reveals the dependency of $Q^O_1$ from the system parameters $E_p^O$, $T_p$, $f_c$, $B$ as well as from the interference parameters $P_1$, $B_1$, $f_1$. In addition, concerning the special case $B_1 = 0$, e.g., a cosine tone, $r_{\nu}$ has to be replaced with $r_{\nu} = \lim_{B_1 \to 0} r_{\nu} = \nu \Delta_{f,k}^{-1}$. Note that this result is consistent to [9] if $P_N = 0$.

The second order moment of the noise and interference-only part $x_{jn}^O$ can be described as

$$Q^O_2 = \int_0^{T_p} \int_0^{T_p} [4E(J(t_1)N(t_1)J(t_2)N(t_2))$$

$$+ 2E(J^2(t_1)N^2(t_2)) + E(N^2(t_1)N^2(t_2)) + E(J^2(t_1)J^2(t_2))] dt_1 dt_2$$

$$= \int_0^{T_p} \int_0^{T_p} [(P_N + P_2)^2 + 2(R_N(\tau) + R_1(\tau))^2] dt_1 dt_2,$$

where $\tau = t_1 - t_2$. Thereby, using the theorem of Price [10], (20) can be written in terms of the noise and interference related autocorrelation functions. With Parseval and the assumptions $2f_c \gg B$, $2f_1 \gg B_1$ and $|f_c + f_1| \gg (B_1 \lor (B - B_1))$ (20) results in

$$Q^O_2 = 2T_p^2 \left[ P_1^2 + P_1 P_N + P_2^2 \right]$$

$$+ \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi f_c T_p)^{2k}}{(2k+1)! (2k+1) (k+1)}$$

$$+ \sum_{k=2}^{\infty} \frac{(-1)^k (2\pi f_c T_p)^{2k-2}}{(2k-2)!} \left( P_1^2 B_2^{2k-2} + P_N^2 B_2^{2k-2} \right)$$

$$+ \frac{2P_1 P_N}{\pi B} \sum_{k=0}^{\infty} \frac{(-1)^k (2\pi f_c T_p)^{2k+2}}{(2k+2)!} \left( \frac{f_{1,k}}{2k+2} + \frac{f_{2,k}}{2k+1} \right). \quad (21)$$

Thereby, with $f_p = \frac{B}{2} + \frac{B_1}{2}$ and $f_m = \frac{B}{2} - \frac{B_1}{2}$, $f_{1,k}$ and $f_{2,k}$
are defined as:

\[
\begin{align*}
    f_{1,k} &= \left( -f_m - \Delta f_{ij} \right)^{2k+2} - \left( -f_p - \Delta f_{ij} \right)^{2k+2} \\
    &+ \left( -f_m + \Delta f_{ij} \right)^{2k+2} - \left( -f_p + \Delta f_{ij} \right)^{2k+2} \\
    f_{2,k} &= \left( f_p + \Delta f_{ij} \right) \left( \left( -f_m - \Delta f_{ij} \right)^{2k+1} - \left( -f_p - \Delta f_{ij} \right)^{2k+1} \right) \\
    &+ \left( f_p - \Delta f_{ij} \right) \left( \left( -f_m + \Delta f_{ij} \right)^{2k+1} - \left( -f_p + \Delta f_{ij} \right)^{2k+1} \right) \\
    &+ \left( f_p - f_m \right) \left( f_m - \Delta f_{ij} \right)^{2k+1} - \left( f_m - \Delta f_{ij} \right)^{2k+1}.
\end{align*}
\]

\(Q_2^O\) is influenced by the system parameters \(T_p, f_c, B\) as well as by the interference parameters \(P_i, B_i, f_i\). However, in contrast to \(Q_1^O\) it cannot be reduced via \(E_P^B\). Eq. (21) simplifies for \(B_1 \to 0\) due to \(P_1^B B_1^2 = P_3^B B_3^2 = 0\), \(f_1 / f_p - f_m = (2k + 2) \left[ \left( -f_c - \Delta f_{ij} \right)^{2k+1} + \left( -f_c + \Delta f_{ij} \right)^{2k+1} \right]\) as well as \(f_1 / f_p - f_m = (2k + 1) \left( f_c + \Delta f_{ij} \right) - \left( f_c - \Delta f_{ij} \right)^{2k}\). Assuming \(P_N = 0\) for this case, (21) equals the result of [9].

**BPPM:** Considering BPPM the second order moment of the signal-noise and signal-interference part \(x_{jn}^P\) is:

\(Q_1^P = \frac{1}{2}Q_1^O\). \(Q_1^P\) differs from \(Q_1^O\) only in a factor of two which can be ascribed to the reduced modulation specific pulse energy. In contrast to \(Q_1^P\) there is a significant difference concerning the second order moment of the noise and interference-only part \(x_{jn}^P\). With the theorem of Price this can be again generally described in terms of the noise and interference specific autocorrelation functions: \((\tau = t_1 - t_2)\)

\[
Q_2^P = 2 \left[ T_p T_s \right] \int_0^{T_p} \int_0^{T_s} \left[ \left( J^2 (t_1) J^2 (t_2) \right) + 4 \left( J (t_1) N (t_2) \right) \left( J (t_2) N (t_1) \right) - 4 \left( J (t_1) N (t_1) \right) \left( J (t_2) N (t_2) \right) \right] dt_1 dt_2 \]

Therefore, using the theorem of Parseval for \(2f_j \gg B_j\), the closed-form result

\[
Q_2^P = 2 \sum_{k=1}^{\infty} \left( -1 \right)^k \left( 2 \right)^{2k} \left( P_2^B B_2^k + P_3^B B_3^{k-2} \right) \frac{g_{2k+2}}{(2k+1)! (2k+1) (k+1)} \]

\[
+ \sum_{k=2}^{\infty} \left( -1 \right)^k \left( 2 \right)^{2k} \left( P_2^B B_2^{k-2} + P_3^B B_3^{k-2} \right) \frac{g_{2k+2}}{(2k)! (2k+2)!} \]

\[
\times \left( \frac{f_{1,k}}{2k+2} + \frac{f_{2,k}}{2k+1} \right),
\]

with

\[
g_{2,\nu} = 2T_{\nu}^* - \left( T_p - \frac{T_{\nu}}{2} \right)^{\nu} + 2 \left( \frac{T_h}{2} \right)^{\nu} - \left( T_p + \frac{T_h}{2} \right)^{\nu},
\]

**IV. RESULTS**

Based on the previous analysis this section identifies the interference robustness of an OOK/BPPM based energy detection receiver. Thereby, assuming regulation of ECC [2] an MIR UWB system with four subbands of equal bandwidth \(B = 625 MHz\) is taken into account. Without loss of generality, the analysis focuses solely on the first subband located at \(f_c = 6.3125 GHz\). However, an extension to other subbands or other MIR UWB system configurations, which are possibly based on other frequency masks, e.g., FCC [1], is easily possible. Further common system parameters used in the subsequent analysis are the pulse duration \(T_p = 3.2 ns\), a duty cycle \(d_s = \frac{1}{B}\), a mean transmit power normalized to one, the modulation specific pulse energy \(E_p^i, i \in \{O, P\}\) as well as a constant signal-to-noise ratio (SNR) of 10 dB at the input of the energy detector. Fixed interference parameter is the interference bit duration \(T_{b,1} = 16T_h = 102.4 ns\).

In Fig. 1 the PG is plotted vs. the SINR_{in}. An interference source with the two bandwidths \(B_{1,1} = 20 MHz\) and \(B_{1,2} = 400 MHz\) is considered leading to the fixed duty cycles \(d_{1,1} = 0.04883\) and \(d_{1,2} = 0.0244\). For OOK/BPPM, the PG increases with higher SINR_{in} up to the interference-free PG at SINR = 10 dB. Furthermore, it can be observed that the OOK/BPPM based PG varies with the interference bandwidth. For OOK, the PG increases with a larger interference bandwidth because of the minor impact of the mixed signal-interference as well as the interference-only component involved in the energy detection. A PG of energy detection can be achieved from a SINR_{in} = -3.5 dB (\(B_{1,1} = 20 MHz\)) and from SINR_{in} = -5.5 dB (\(B_{1,2} = 400 MHz\)), respectively. For strong narrow- and broadband interference
no PG results as the energy detector’s decision variable (8) is significantly corrupted. In contrast, considering BPPM a PG can be achieved for small interference bandwidths, e.g., $B_{1,1} = 20 \text{ MHz}$, over nearly the complete SINR$_{in}$ range. For $B_{1,2} = 400 \text{ MHz}$ a PG occurs from SINR$_{in} = -2 \text{ dB}$. The reason for this behavior lies in a different amount of energy resulting from the mixed signal-interference and interference-only term within the two observation periods of duration $T_p$ (12). Finally, considering OOK/BPPM with respect to their relative PG shows that for strong NBI BPPM is more robust whereas OOK is more robust for mean and low interference.

Fig. 2 shows the PG vs. $f_1$, which varies from $f_c - \frac{B}{2}$ to $(f_c + f_1 \pm \frac{B}{2})$. This can be on one hand ascribed to the subband pulse’s sinc spectrum which is zero at the subband’s boundary. On the other hand, the more $f_1$ is located at the subband’s boundary the minor the interference bandwidth falling into the subband. In case interference overlaps with the subband’s boundary, the effective interference parameters $B_1$, $f_1$ and $d_1$ changes resulting in a reduction of the actual mean interference power $P_I$.

V. Conclusion

This paper investigates the energy detector’s robustness in presence of narrow- and broadband interference within an OOK/BPPM based noncoherent MIR UWB communication system. Based on the energy detector’s PG closed-form expressions of noise and interference related second order moment statistics at the output of an energy detection receiver are provided. This reveals insight into the impact of interference and system specific parameters. Furthermore, the analysis of the relative modulation specific PG with respect to various interference parameters shows the robustness of OOK/BPPM.

Future work focuses on the approach’s extension to other pulse shapes, e.g., cosine-shaped pulses, to other modulation schemes as well as to realistic channel models.

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References


