Optimum phase measurement in the presence of noise

<u>Darko Zibar</u>, Jens E. Pedersen, Poul Varming, Francesco Da Ros

Machine Learning in Photonic Systems (M-LiPS) group
DTU Fotonik, Department of photonics Engineering
Technical university of Denmark
Email: dazi@fotonik.dtu.dk

Research question to be answered

What is the impact of amplifier noise on signal phase?

- Relevant for building high-power ultra-narrow linewidth lasers
- Relevant for generation of optical frequency combs
- Relevant for transmission of frequency standards
- Relevant for noise characterization of lasers and frequency combs

Surprisingly few works on the topic and no common agreement

SPECTRAL BROADENING DUE TO FIBRE AMPLIFIER PHASE NOISE

ELECTRONICS LETTERS 29th March 1990 Vol. 26 No. 7

G. J. COWLE†
P. R. MORKEL
R. I. LAMING
D. N. PAYNE

1554 IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 34, NO. 9, SEPTEMBER 1998

Novel Aspects of Spectral Broadening Due to Fiber Amplifier Phase Noise

Lothar Möller

IEEE JOURNAL ON SELECTED TOPICS IN QUANTUM ELECTRONICS, VOL. 7, NO. 1, JANUARY/FEBRUARY 2001 49

New Investigations on the Effect of Fiber Amplifier Phase Noise

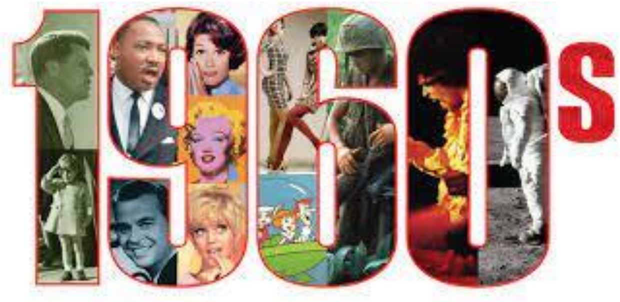
Etienne Rochat and René Dändliker

2610 Vol. 59, No. 8 / 10 March 2020 / Applied Optics Research Article

applied optics

Influence of amplified spontaneous emission on laser linewidth in a fiber amplifier

MINGYUAN XUE,[1,2] CUNXIAO GAO,[1,*] LINQUAN NIU,[1] SHAOLAN ZHU,[1] AND CHUANDONG SUN[1]



1604 *PROCEEDINGS OF THE IRE* *July*

The Fundamental Noise Limit of Linear Amplifiers*

H. HEFFNER†, FELLOW, IRE

* Received January 8, 1962; revised manuscript received, April 30, 1962.

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Quantum Noise in Linear Amplifiers

H. A. HAUS
Electrical Engineering Department and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts

AND

J. A. MULLEN
Research Division, Raytheon Company, Waltham, Massachusetts

(Received May 10, 1962; revised manuscript received August 23, 1962)

Both papers derive that *power* and *phase fluctuation* due to amplifier noise are given by:

$$\Delta P = (G-1)h\nu B$$

$$\Delta\phi = \frac{(G-1)h\nu B}{2P}$$

Heffner and Haus assume input to the amplifier contains single frequency
(Lasers have amplitude and phase noise)

Assumptions by Heffner and Haus

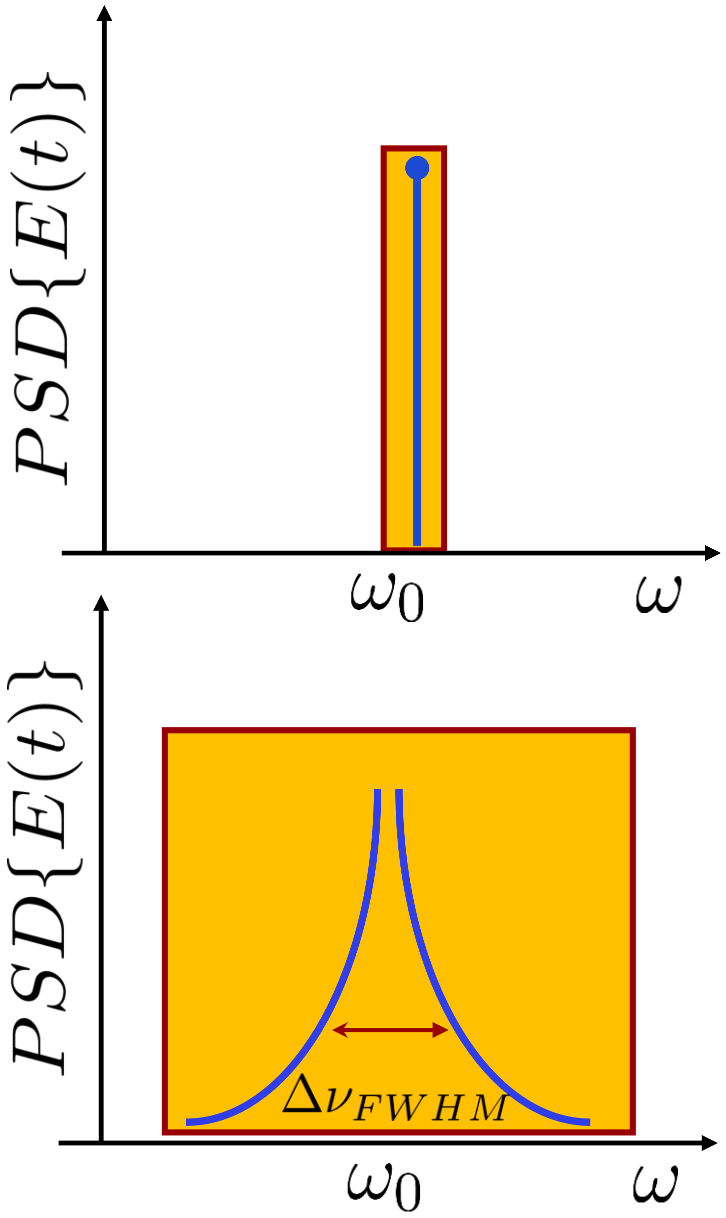
Constant phase signal:

$$E(t) = \sqrt{P_0}\sin[\omega_0 t + \phi_0]$$

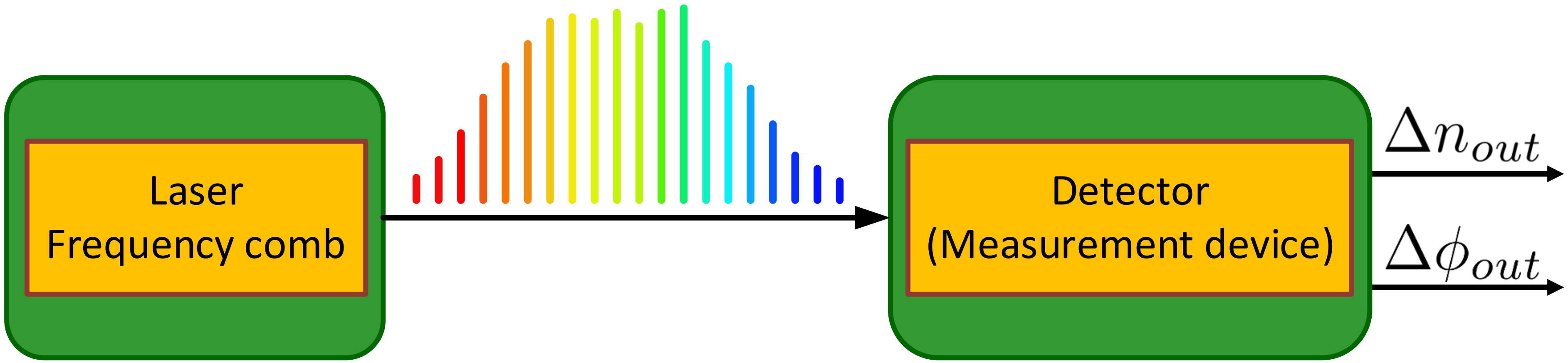
In practice amplitude and phase are (randomly) time-varying :

$$E(t) = \sqrt{P_0(1+\alpha(t))}\sin[\omega_0 t + \phi_0(t)]$$

The implication of time-varying amplitude and phase:
Measurement bandwidth needs to be carefully chosen



Ultimate laser stability conditioned by measurement precision



Heisenberg uncertainty sets limit on how accurately *photon number* and *phase* can be measured:

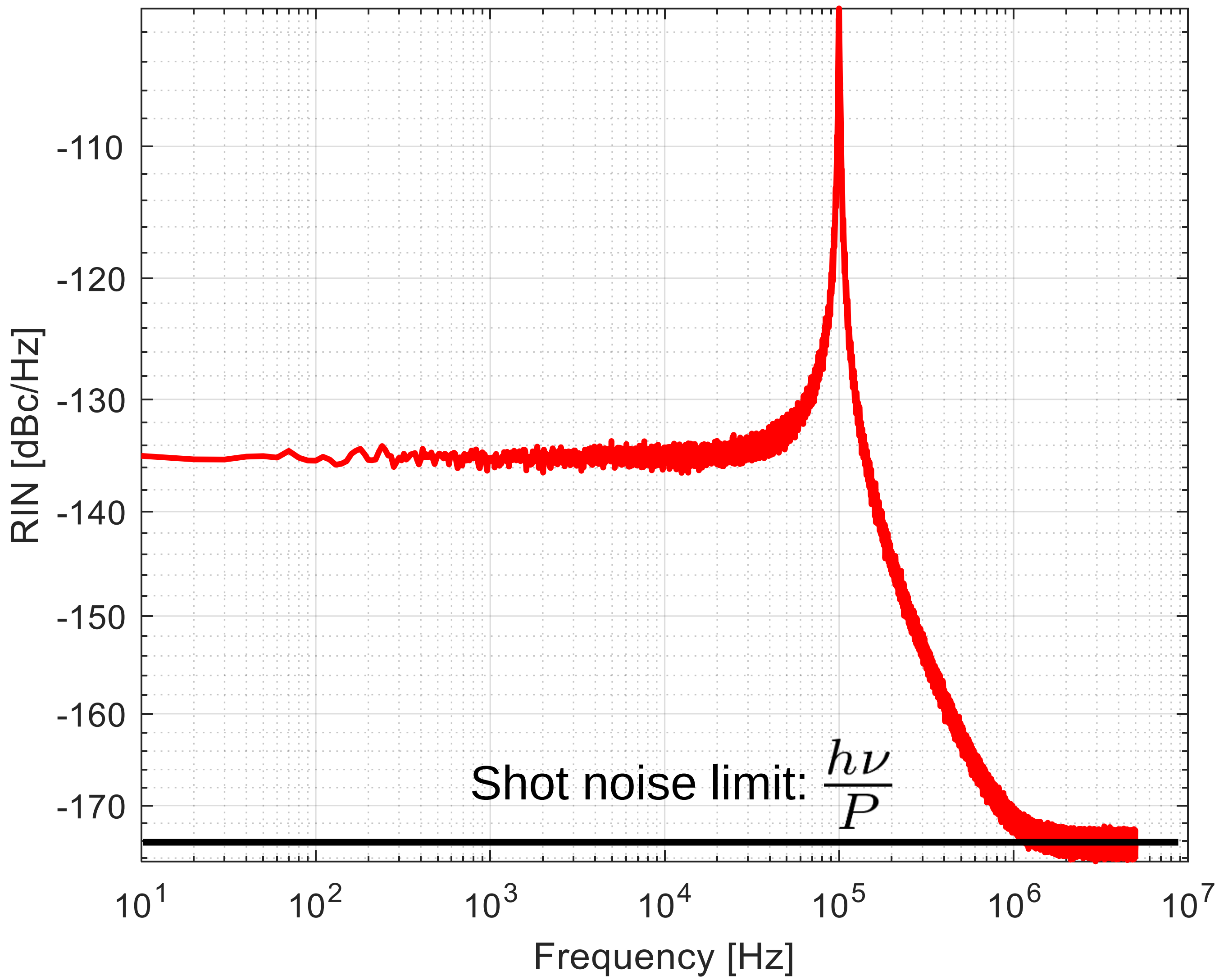
$$\Delta n_{out} \Delta \phi_{out} \geq \frac{1}{2}$$

In spectral domain *Heisenberg uncertainty* translates to:

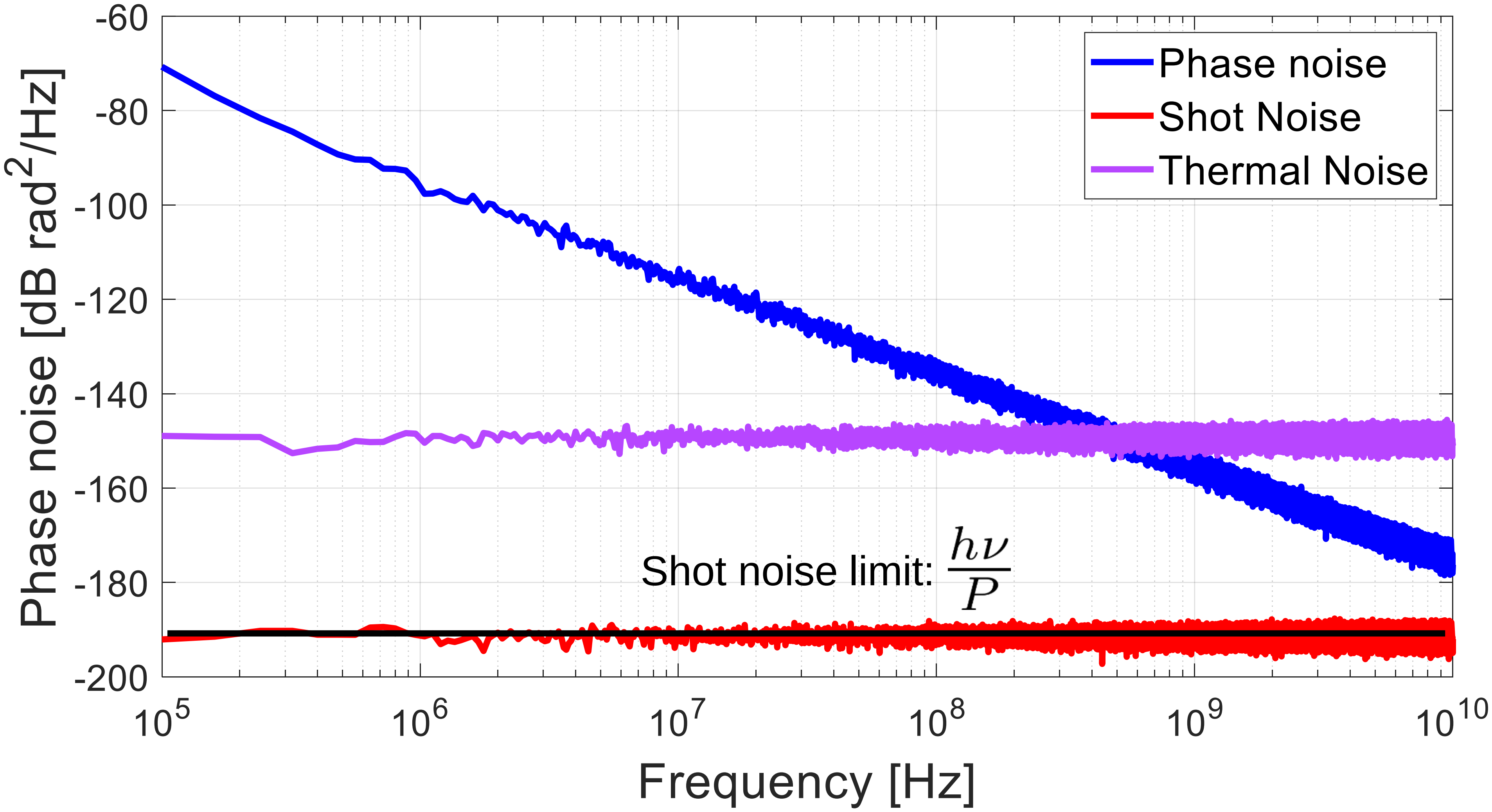
$$S_{RIN}(f) S_{\phi}(f) \geq \left(\frac{h\nu}{2P}\right)^2$$

A. Yariv, SPIE 1995

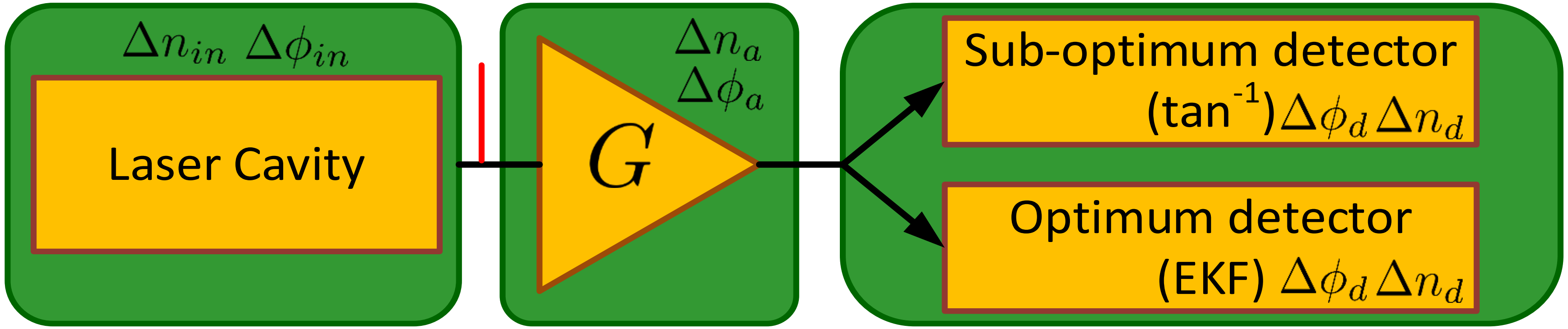
Limit on measuring Relative Intensity Noise (RIN)



Limit on measuring phase noise



The concept of optimum detector



$$\Delta n_{out}^2 = \Delta n_a^2 + \Delta n_d^2$$

$$\Delta \phi_{out}^2 = \Delta \phi_a^2 + \Delta \phi_d^2$$

Ultimate performance limit governed by:

$$\Delta n_{out} \Delta \phi_{out} \geq \frac{1}{2}$$

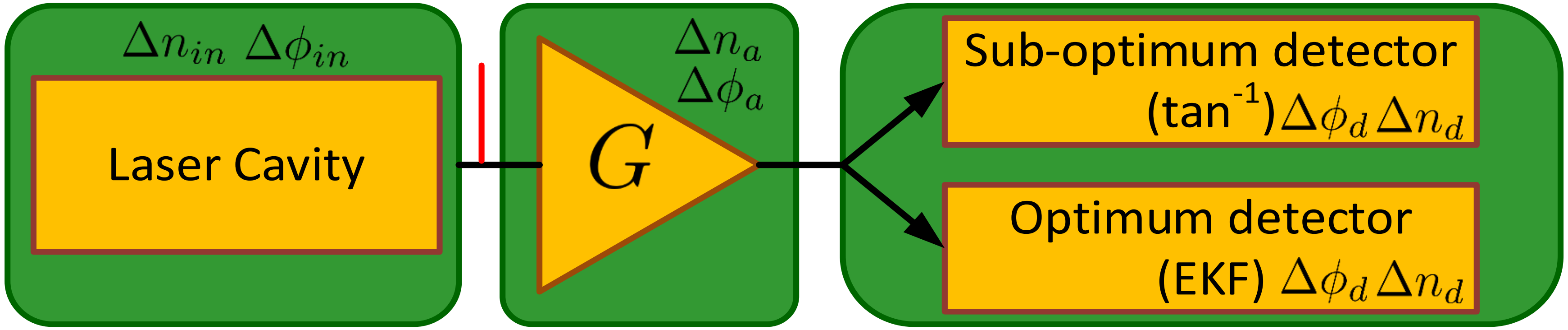
Heisenberg uncertainty limit reached when:

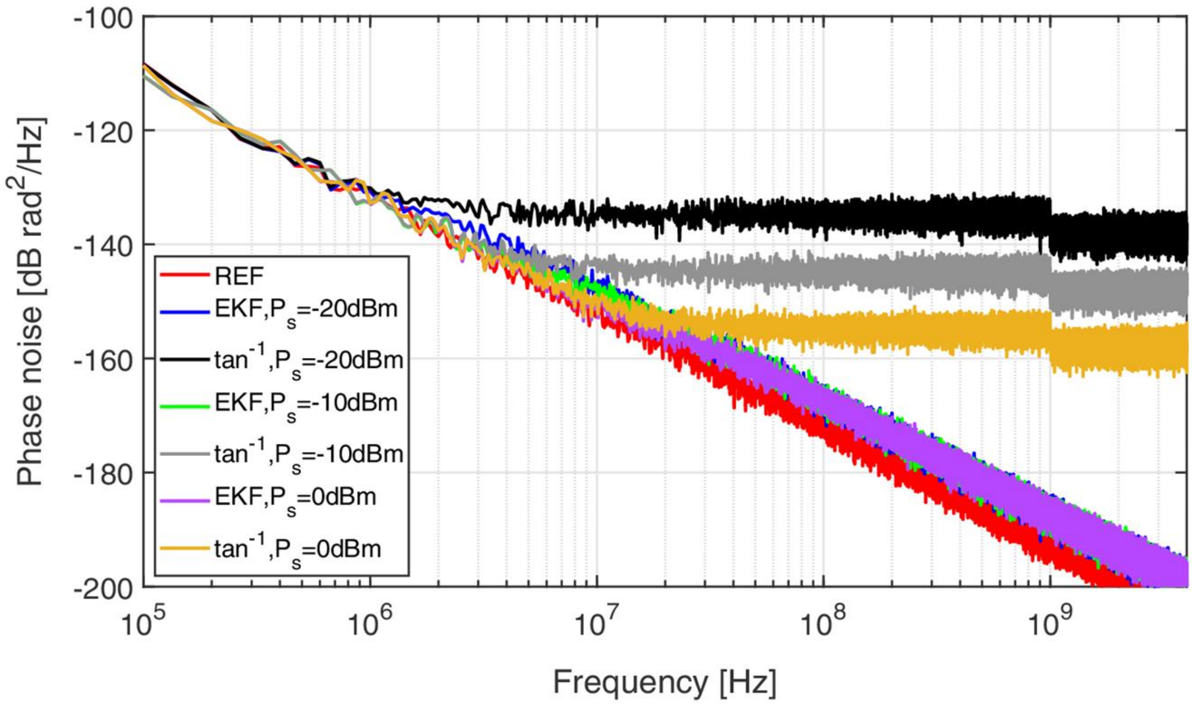
$$\frac{\Delta n_d}{\Delta \phi_d} = \frac{\Delta n_a}{\Delta \phi_a}$$

Detector uncertainty needs to be *matched* to amplifier uncertainty (optimum detector)

H. Heffner, 1962

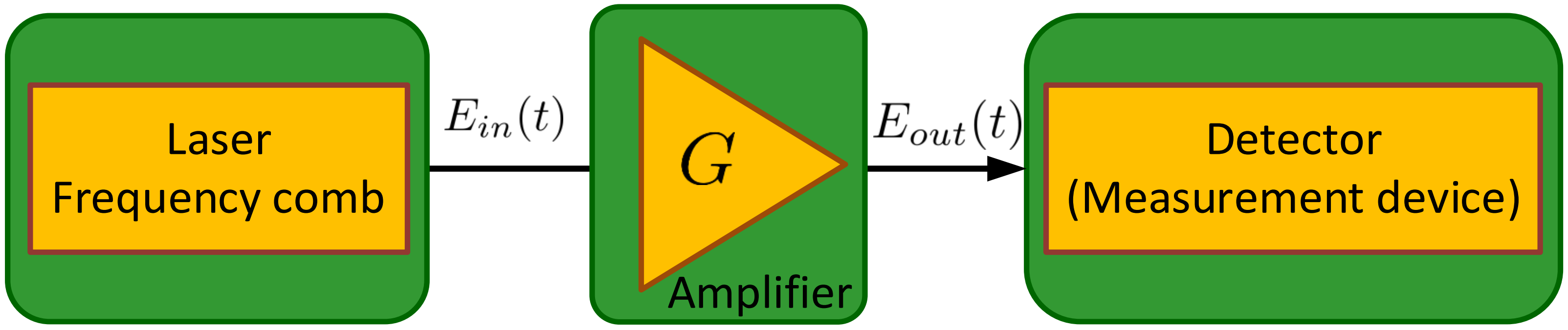
The importance of optimum detector





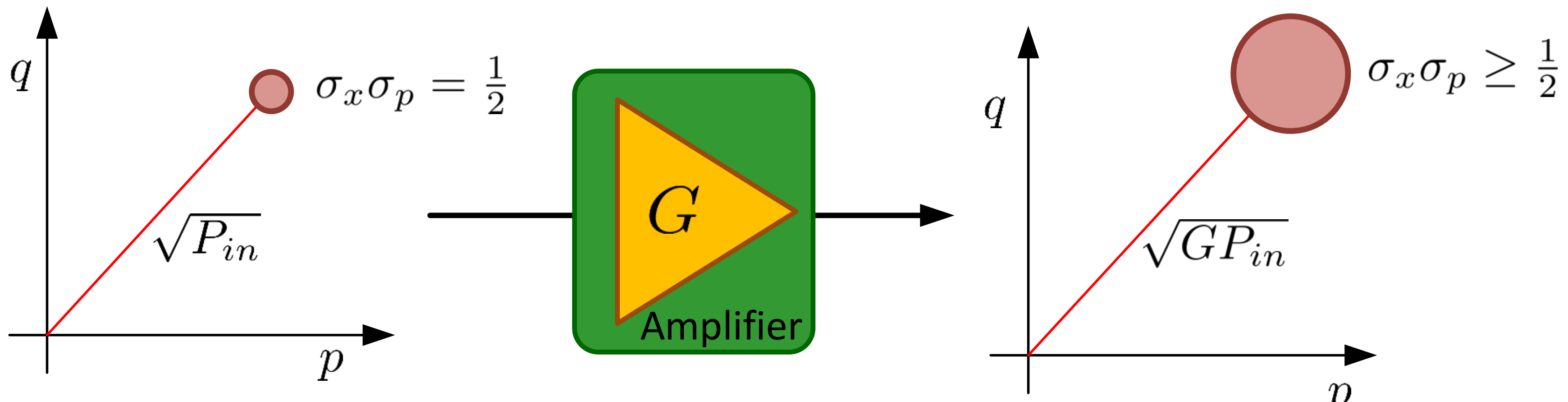
Darko Zibar, Jens E. Pedersen, Poul Varming, Giovanni Brajato, Francesco Da Ros, "Approaching optimum phase measurement in the presence of amplifier noise," Optica **8,** 1262-1267 (2021);

Minimum noise added by the amplifier



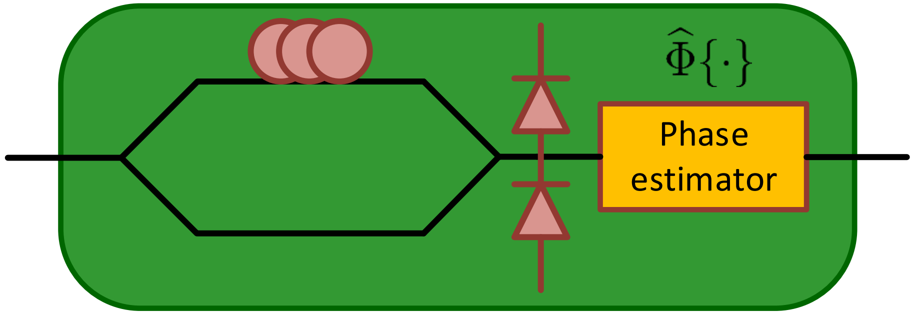
$$E_{in}(t) = \sqrt{P_{in}}\sin[\omega t + \phi_0] \qquad E_{out}(t) = \sqrt{GP_{in}}\sin[\omega t + \phi_0] + N(t)$$

$N(t)$: Gaussian noise with zero mean and standard deviation $\Delta P = (G-1)h\nu B$

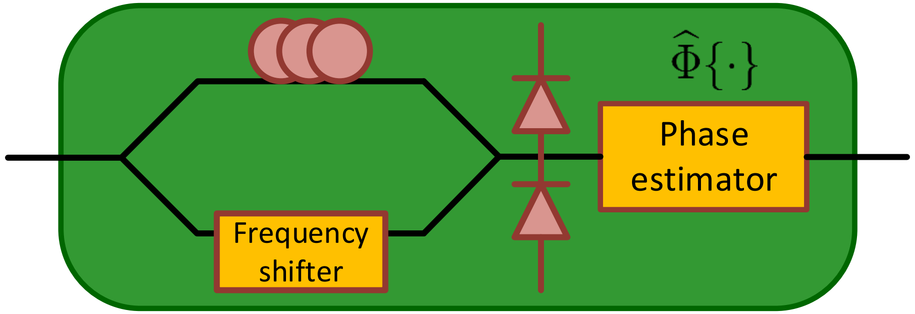


E. Desurvire, 1994

Quantum limited phase detection (the best we can do)

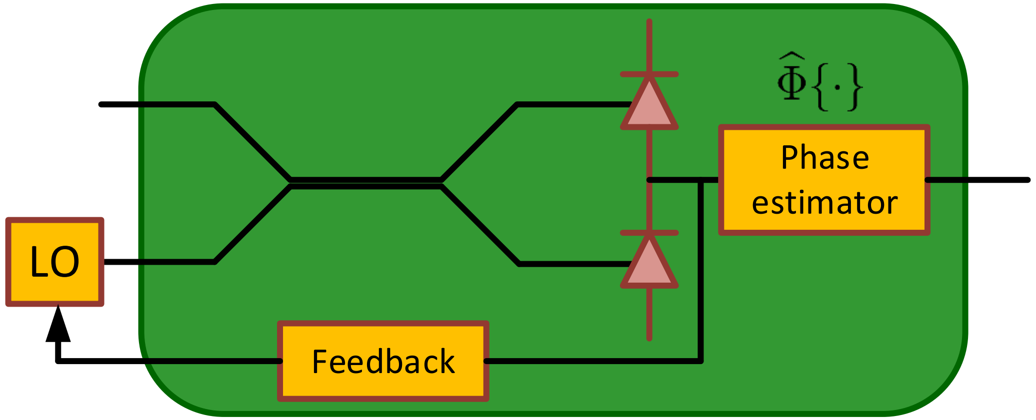


$$\Delta\phi_{ql}^2 = \frac{1}{\sqrt{N_p}}$$

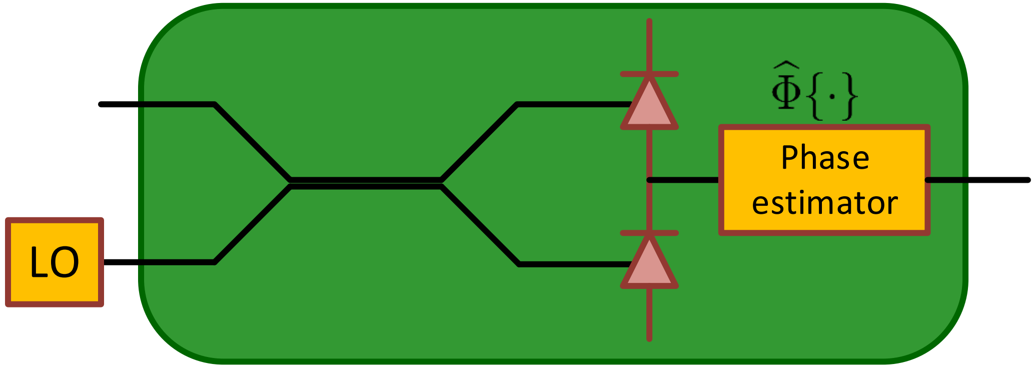


$$\Delta\phi_{ql}^2 = \frac{1}{\sqrt{N_p/2}}$$

$$N_p = \frac{P_{in}}{h\nu\pi\Delta\nu_{FWHM}}$$



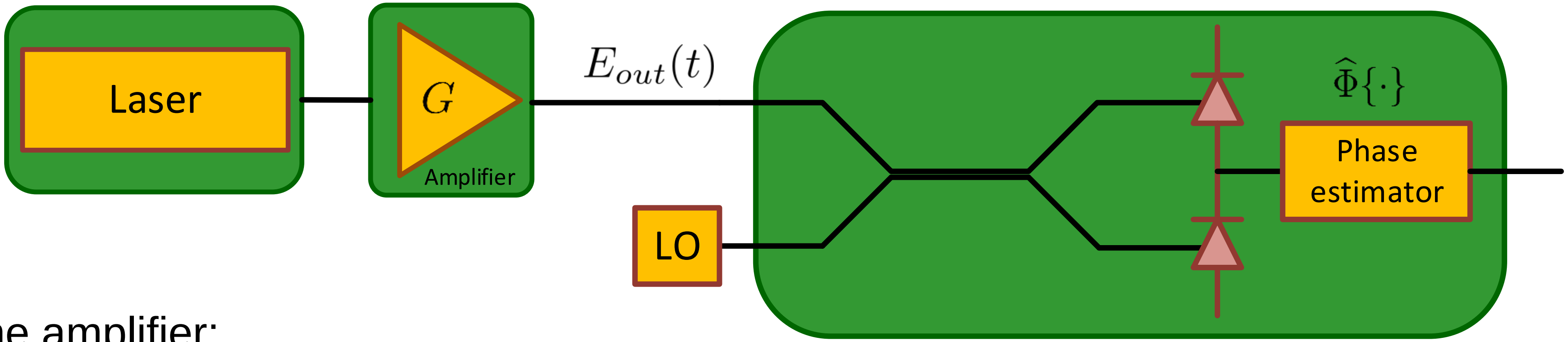
$$\Delta\phi_{ql}^2 = \frac{1}{2\sqrt{N_p}}$$



$$\Delta\phi_{ql}^2 = \frac{1}{2\sqrt{N_p/2}}$$

H. M . Wiseman, 2002

Minimum phase fluctuation due to amplifier noise



Output of the amplifier:

$$E_{out}(t) = \sqrt{GP_{in}}\sin[\omega_0 t + \phi(t)] + n_a(t)$$

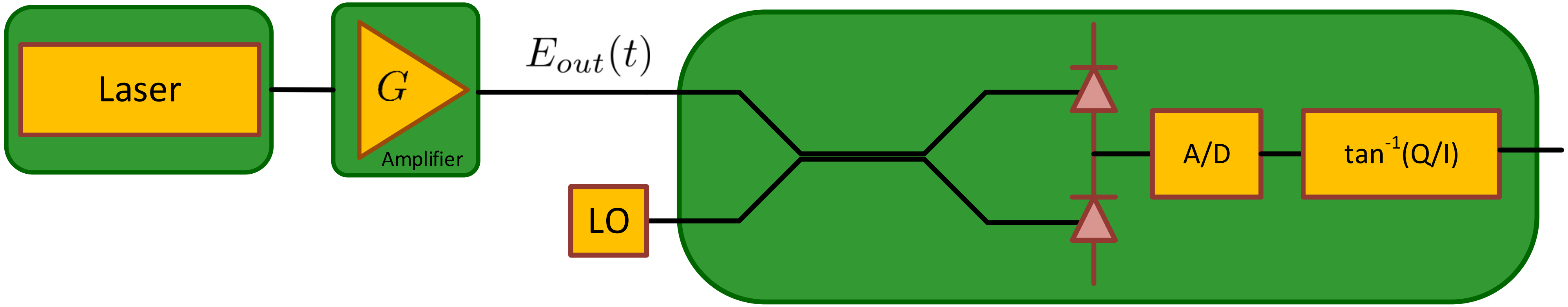
$n_a(t)$: Gaussian noise term added by the amplifier with variance :

$$\sigma_N^2 = P_N = h\nu(G-1)B$$

Quantum limited (minimum) phase fluctuation due to amplifier noise $(B = \Delta\nu_{FWHM})$:

$$\Delta\phi_a^{MAP} = \frac{1}{2\sqrt{\sqrt{N_p/2}}} = \frac{1}{\sqrt{2\sqrt{GP_{in}/2h\nu(G-1)\pi\Delta\nu_{FWHM}}}}$$

Optimum phase measurement for *high* SNR



Discrete-time signal after analogue-to-digital converter (A/D):

$$y[k] = 2R\sqrt{GP_{in}P_{LO}}\cos(\Delta\omega kT_s + \phi[k]) + n^{sh}[k] + n^b[k]$$

Phase estimation method: $\phi_{\tan^{-1}} = \arg[(y[k] + j\mathcal{H}\{y[k]\})e^{-j\Delta\omega t}] = \arg[(I + jQ)e^{-j\omega t}] = \tan^{-1}(Q/I)$

Quantum limited (minimum) phase fluctuation due to amplifier noise:

$$\Delta\phi_a = \frac{1}{2\sqrt{N_p/2}} = \frac{1}{2\sqrt{GP_{in}/2h\nu(G-1)B}} = \frac{1}{2\sqrt{SNR/2}}$$

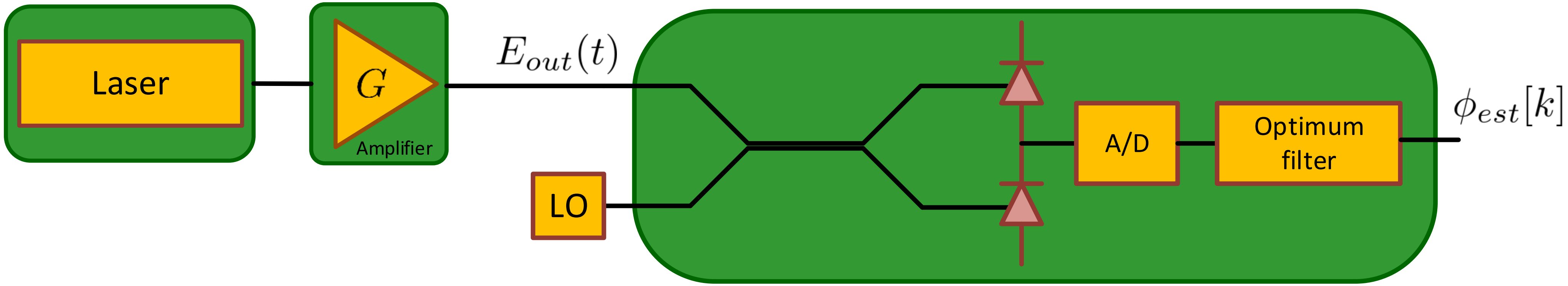
Measuring at low and medium SNR is important

- Technical noise dominates laser phase noise at low frequencies
- Determining fundamental laser linewidth (quantum noise limited) requires measuring beyond MHz
- Laser power may be low (output of the cavity)
- Frequency comb lines may have low power
- Several stages of amplification may reduce SNR

Signal-to-noise ratio of beat signal after heterodyne detection:

$$SNR = \frac{2RP_sP_{LO}}{\sigma^2_{shot}+\sigma^2_b} = \frac{2RP_sP_{LO}}{2qRP_{LO}B+4P_{LO}N_Ah\nu(G-1)B}$$

Optimum filtering for *wide-range* of SNRs



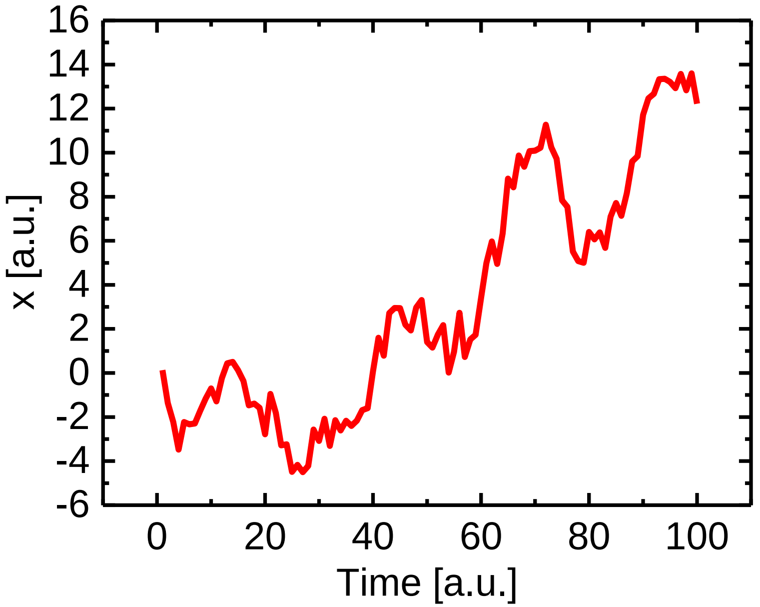
Given discrete-time signal after analogue-to-digital converter (A/D):

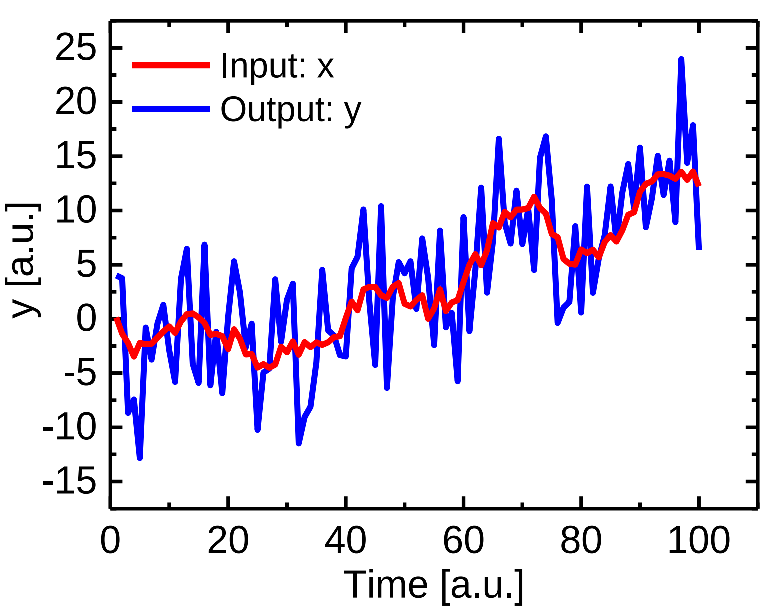
$$y[k] = 2R\sqrt{GP_{in}P_{LO}}\cos(\Delta\omega kT_s + \phi[k]) + n^{sh}[k] + n^{b}[k]$$

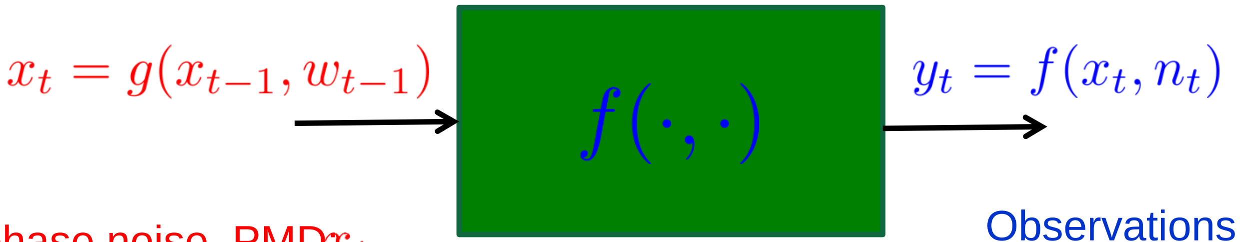
Optimum filter finds phase that is closest to $\phi[k]$ for a given SNR

For extracting signal from noise Bayesian filter is theoretically optimum filter
Kalman filter is an approximation of Bayesian filter

Bayesian filtering: infer *input x* from *noisy output y*







States: amp., phase noise, PMD: x_t

Mapping function: $g(\cdot)$

Process noise: w_{t-1}

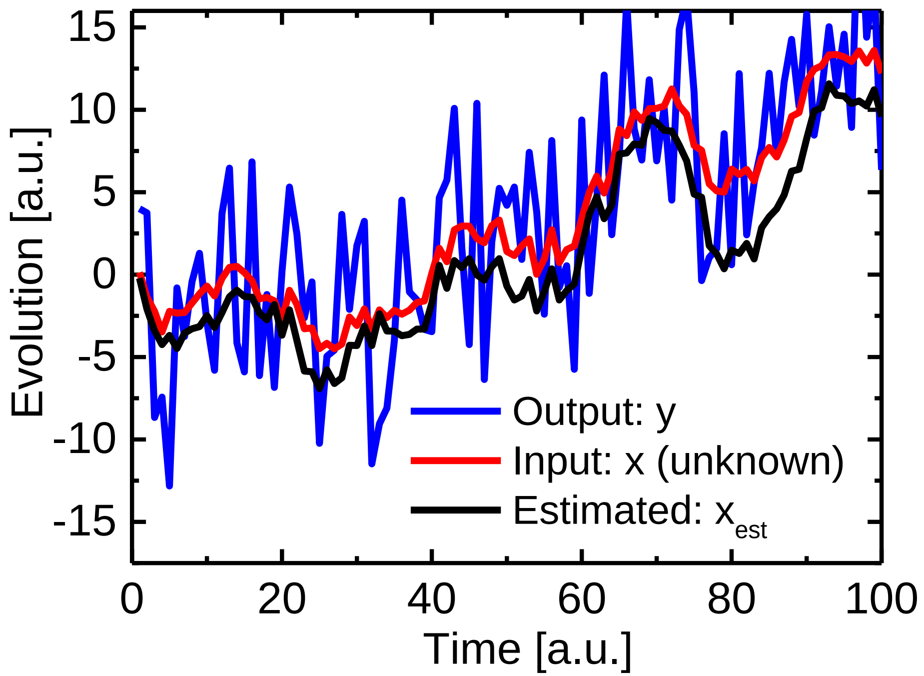
Observations: y_t

Mapping function: $f(\cdot)$

Measurement noise: n_t

S. Sarka, 2013

Bayesian filtering: infer *input x* from *noisy output y*



Given a measurement:

$$y = x + n$$

Compute:

$$\overbrace{p(x|y)}^{posterior} = \frac{p(y|x)\,\overbrace{p(x)}^{prior}}{p(y)} \longrightarrow x_{est} = E[x] = \int x p(x|y) dx$$

Bayesian filtering equations

Deterministic state space model:

$$x_t = g(x_{t-1}, w_{t-1})$$
$$y_t = f(x_t, n_t)$$

Probabilistic state space model:

$$\theta \sim p(\theta)$$
$$x_t \sim p(x_t | x_{t-1}, \theta)$$
$$y_t \sim p(y_t | x_t, \theta)$$

Use Kalman or particle filtering to solve:

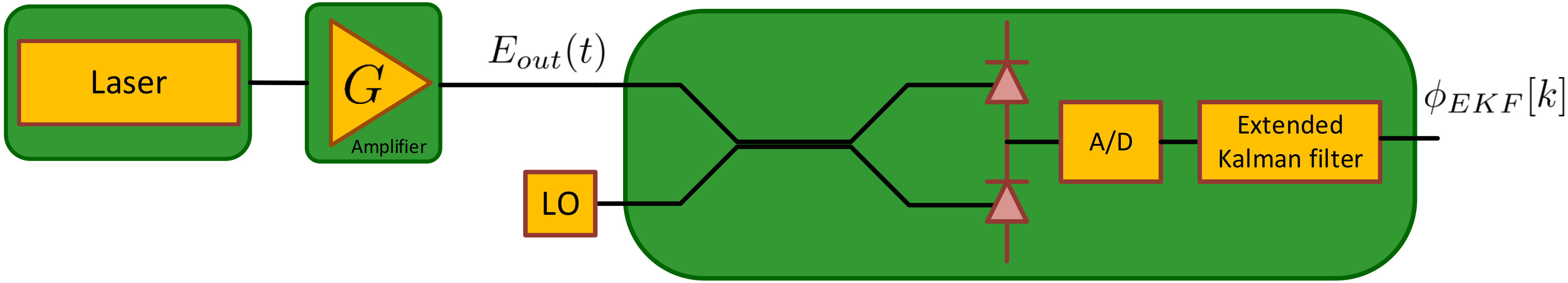
for t=1:T

1. *Compute prior:* $p(x_t|y_{1:t-1}) = \int p(x_t|x_t - 1) p(x_{t-1}|y_{1:t-1}) dx_{t-1}$

2. *Compute posterior:* $p(x_t|y_{1:t}) = \dfrac{p(y_t|x_t, \theta) p(x_t|y_{1:t-1}, \theta)}{p(y_{1:k}|\theta)}$

end

Approaching optimum filtering with Kalman filter



EKF based phase estimation approaches quantum limit:

$$\sigma_{EKF} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\phi^{true}[k] - \phi_{EKF}[k])^2} \rightarrow \Delta\phi^{MAP} = \frac{1}{\sqrt{2\sqrt{N_p/2}}}$$

Practical implication of random phase fluctuations

Given a observation time *T*, laser phase noise variance is expressed as:

$$\Delta\phi^2(T) = 2\pi\Delta\nu T$$

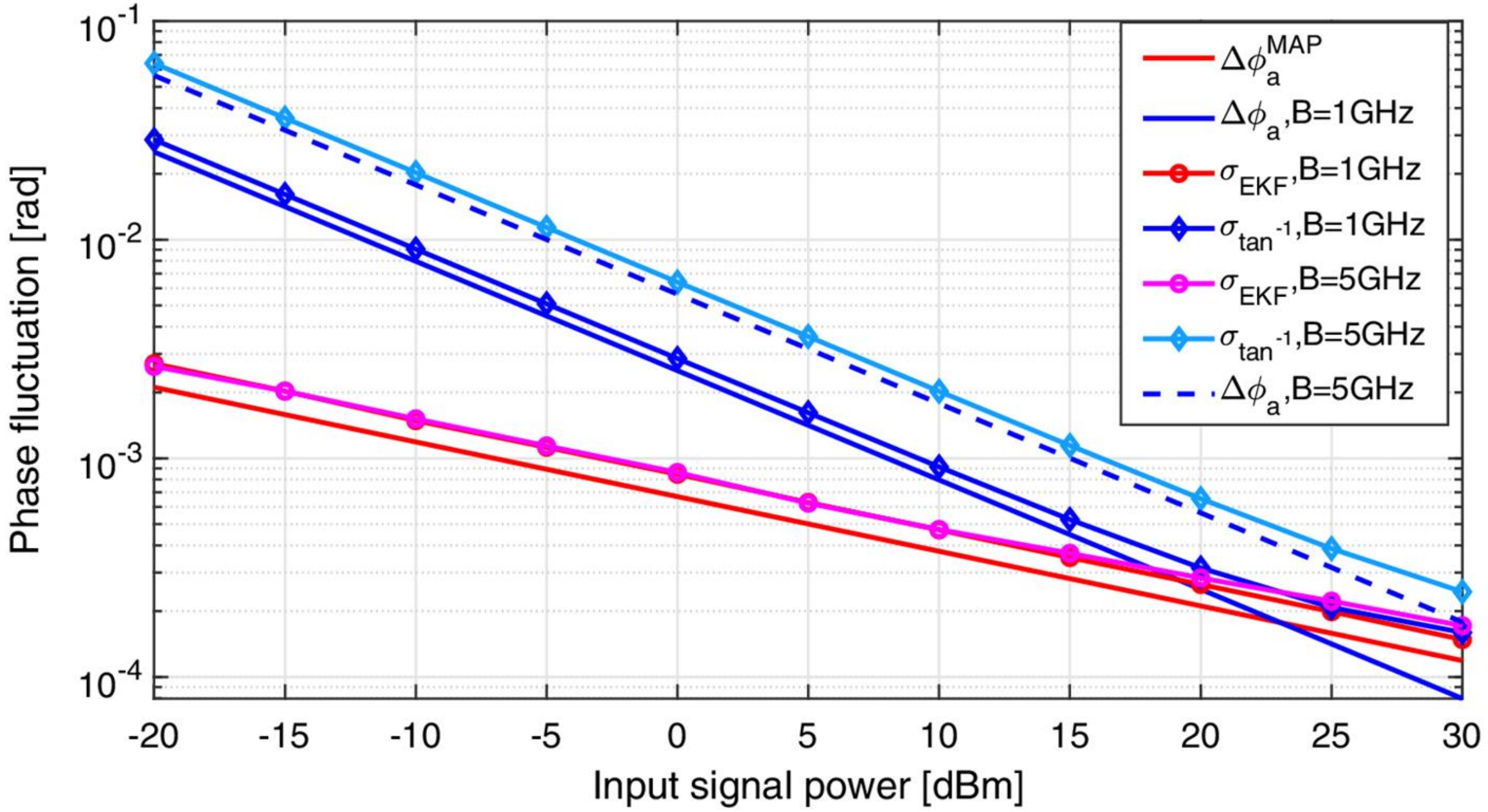
The corresponding *spectral broadening* expressed as:

$$\Delta\nu = \frac{\Delta\phi^2(T)}{2\pi T}$$

The quantum limited spectral broadening due to amplifier noise :

$$\Delta\nu_a^{MAP} = \frac{\Delta\phi_a^{MAP}}{2\pi T}$$

Numerical results: phase fluctuation

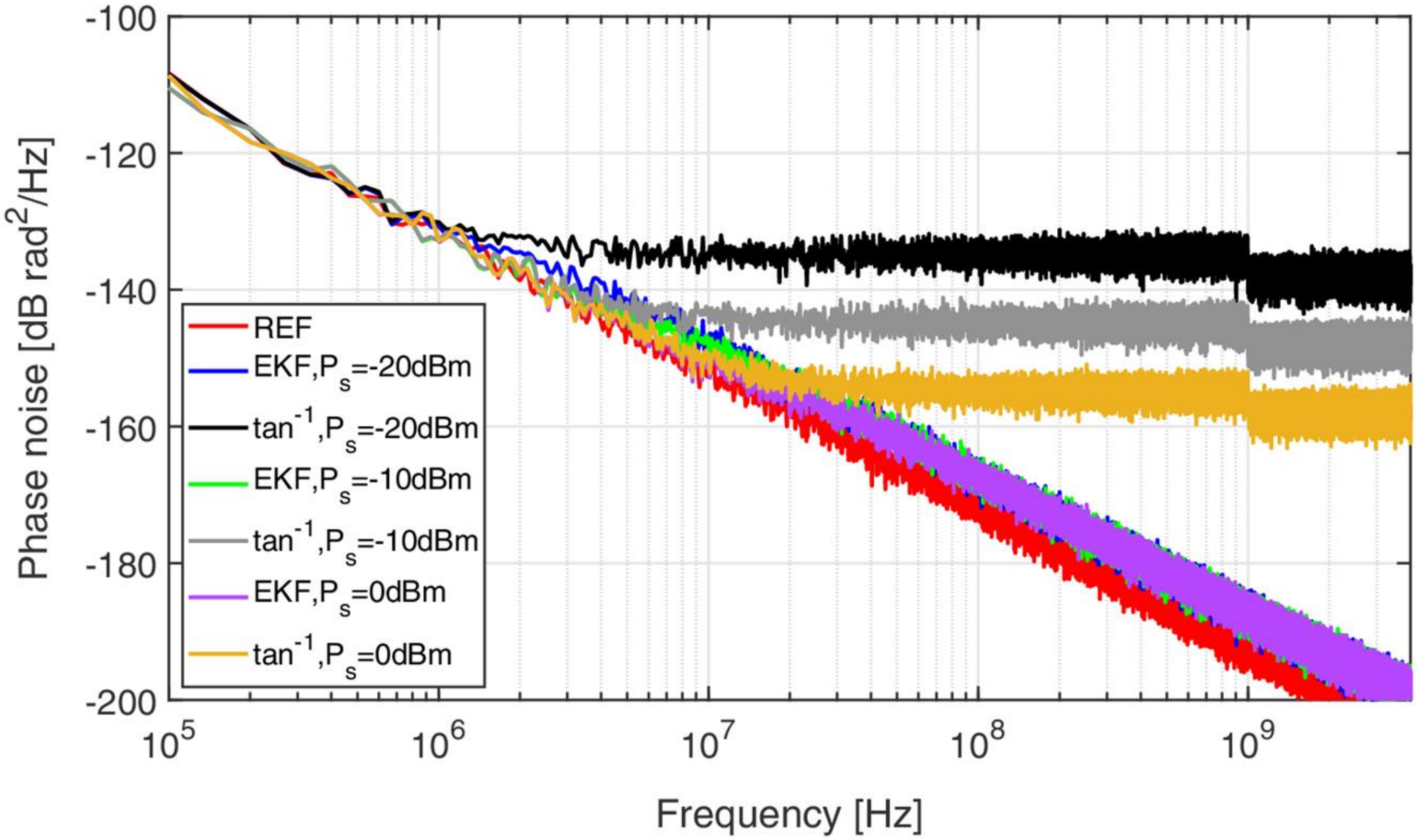


$$\Delta\nu_{FWHM} = 1 \text{ kHz}$$

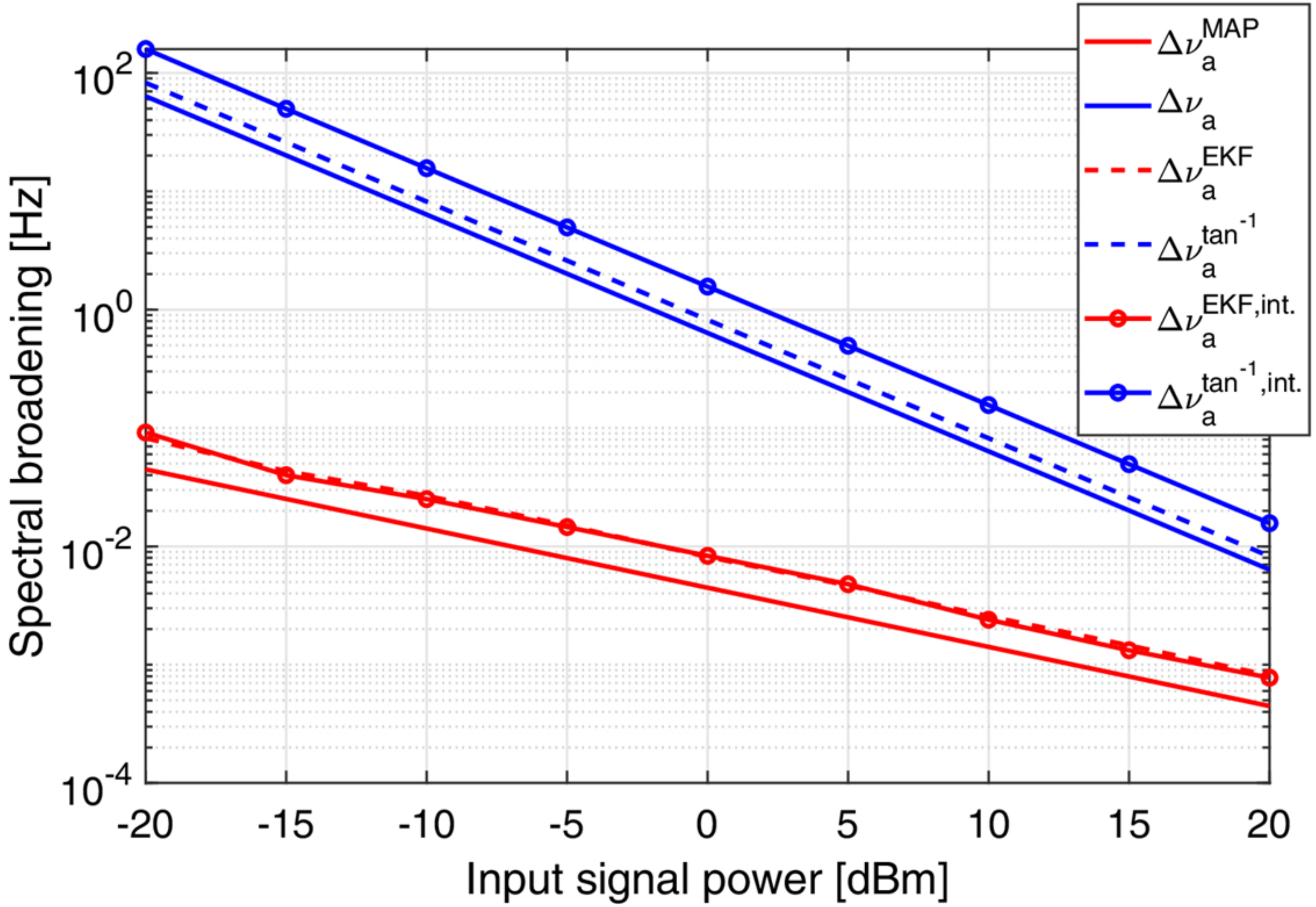
$$\Delta\phi_a^{MAP} = \frac{1}{2\sqrt{\sqrt{N_p/2}}} = \frac{1}{\sqrt{2\sqrt{GP_{in}/2h\nu(G-1)\pi\Delta\nu_{FWHM}}}}$$

$$\Delta\phi_a = \frac{1}{2\sqrt{N_p/2}} = \frac{1}{2\sqrt{GP_{in}/2h\nu(G-1)B}}$$

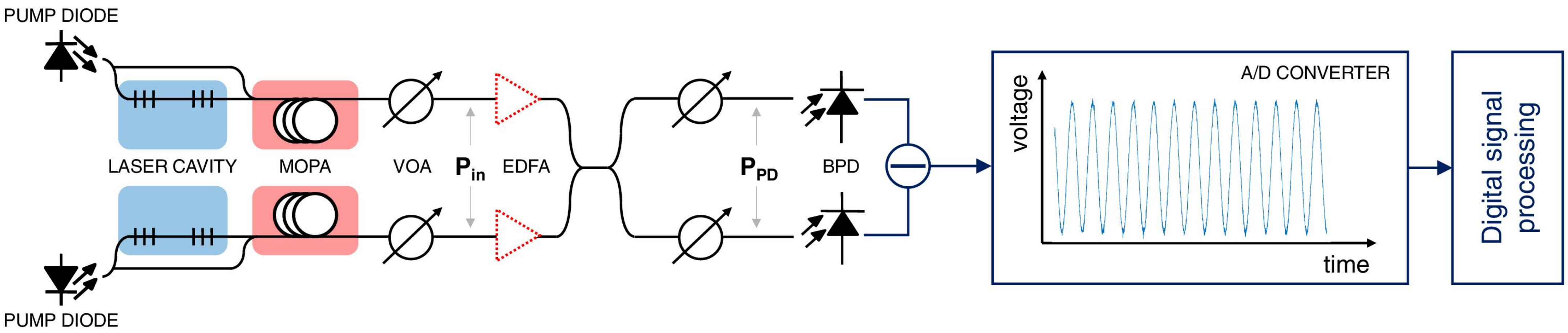
Numerical results: phase PSD



Numerical results: spectral broadening



Experimental set-up



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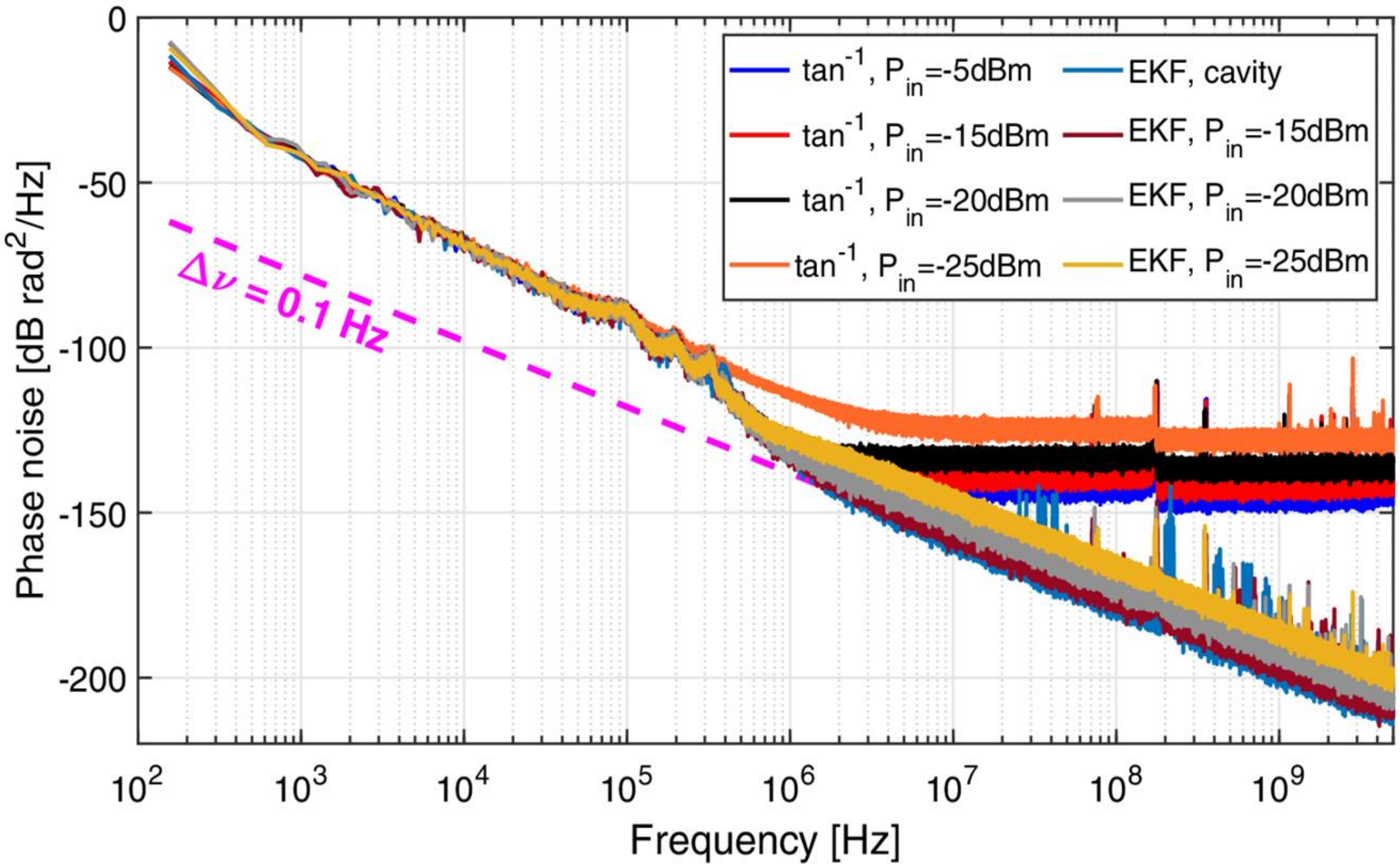
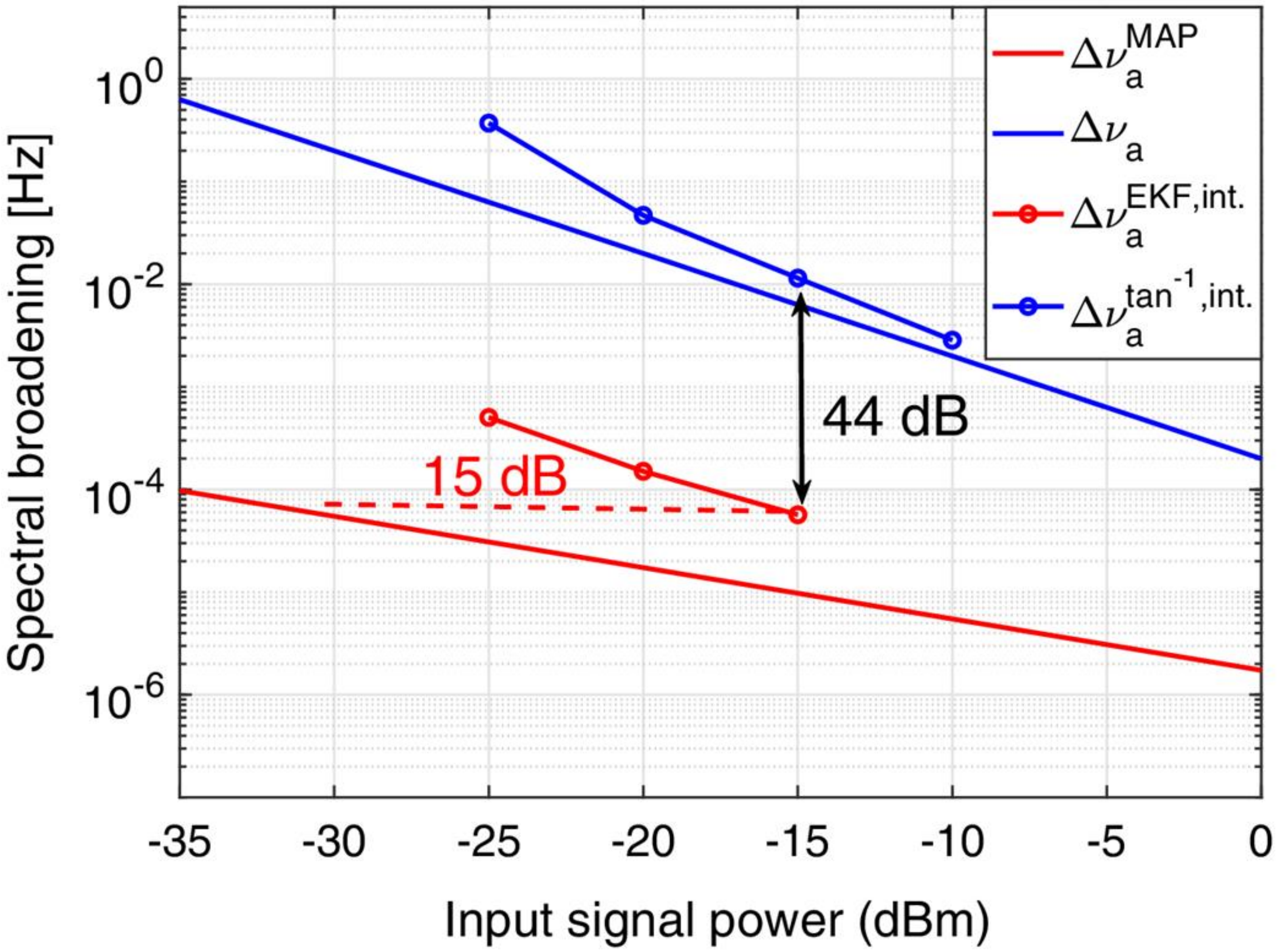
https://www.osapublishing.org/optica/abstract.cfm?uri=optica-8-10-1262

Experimental: phase-noise measurement



Experimental: spectral broadening



Conclusion and outlook

- Impact of amplifier noise on signal phase quantified
- Minimum impact of amplifier noise requires optimum (phase) detector
- Extended Kalman filtering approaches optimum detector
- Importance of optimum detector for measuring laser phase-noise demonstrated
- Measurement noise floor originates form non-optimal detector
- With optimum detector noise floor avoided $\Rightarrow$ fundamental laser linewidth revealed