Physics-Based Machine Learning for Fiber-Optic Communication Systems

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Workshop on Neuromorphic High-Speed Communications (NeuCoS)
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FIBER-OPTIC COMMUNICATIONS RESEARCH CENTER

CHALMERS

Thank You!

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Duke

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Chalmers (now: Ericsson)

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Chalmers

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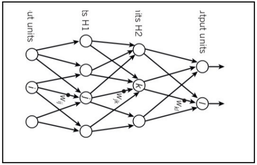
This work started with a simple observation . . .

Deep Learning [LeCun et al., 2015]



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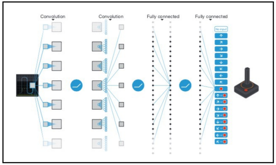

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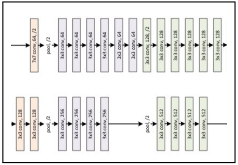

. . .

Multi-layer neural networks: impressive performance, countless applications

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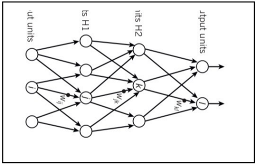

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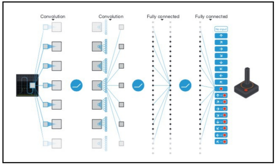

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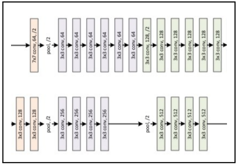

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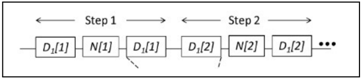

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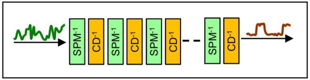

[Nakashima et al., 2017]

Split-step methods for solving the propagation equation in fiber-optics

Agenda

In this talk, we . . .

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1. show that multi-layer neural networks and the split-step method have the same functional form: both alternate linear and pointwise nonlinear steps
2. propose a physics-based machine-learning approach based on parameterizing the split-step method (no black-box neural networks)
3. revisit hardware-efficient nonlinear equalization via learned digital backpropagation

Outline

1. Machine Learning and Neural Networks for Communications
2. Physics-Based Machine Learning for Fiber-Optic Communications
3. Learned Digital Backpropagation
4. Conclusions

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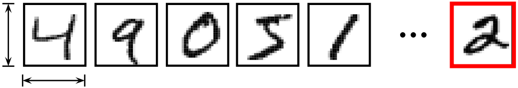
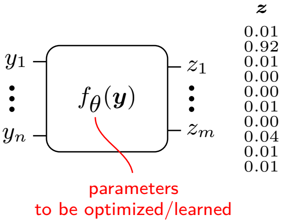
Supervised Learning

handwritten digit recognition (MNIST: 70,000 images)



28×28 pixels $\implies n = 784$



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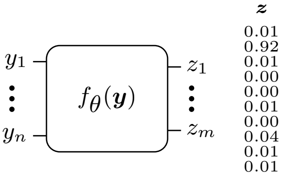
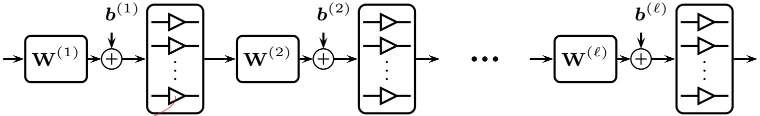
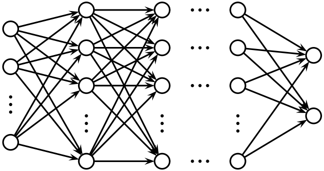


How to choose $f_\theta(\boldsymbol{y})$? Deep feed-forward neural networks



activation function



equivalent graph representation

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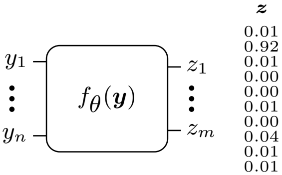

How to optimize $\theta = \{\boldsymbol{W}^{(1)}, \ldots, \boldsymbol{W}^{(\ell)}, \boldsymbol{b}^{(1)}, \ldots, \boldsymbol{b}^{(\ell)}\}$?

Supervised Learning

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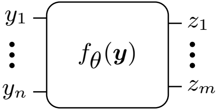


$\boldsymbol{z}$	$\boldsymbol{x}$
0.01	0
0.92	1
0.01	0
0.00	0
0.00	0
0.01	0
0.00	0
0.04	0
0.01	0
0.01	0

How to optimize $\theta = \{\boldsymbol{W}^{(1)}, \dots, \boldsymbol{W}^{(\ell)}, \boldsymbol{b}^{(1)}, \dots, \boldsymbol{b}^{(\ell)}\}$?

Given a data set $\mathcal{D} = \{(\boldsymbol{y}^{(i)}, \boldsymbol{x}^{(i)})\}_{i=1}^N$, where $\boldsymbol{y}^{(i)}$ are model inputs and $\boldsymbol{x}^{(i)}$ are labels, we iteratively minimize

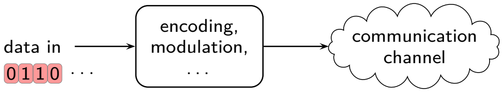
$$\frac{1}{|\mathcal{B}_k|} \sum_{(\boldsymbol{y},\boldsymbol{x}) \in \mathcal{B}_k} \mathcal{L}(f_\theta(\boldsymbol{y}), \boldsymbol{x}) \triangleq g(\theta) \qquad \text{using} \qquad \underbrace{\theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k)}_{\text{stochastic gradient descent}}$$

- $\mathcal{B}_k \subset \mathcal{D}$ and $|\mathcal{B}_k|$ is called the batch (or minibatch) size
- Typical loss function: mean squared error $\mathcal{L}(\boldsymbol{a}, \boldsymbol{b}) = \|\boldsymbol{a} - \boldsymbol{b}\|^2$ (regression)
- λ is called the step size or learning rate

Physical-Layer Design: Conventional vs. Machine Learning

- Conventional: handcrafted DSP blocks based on mathematical modeling

Physical-Layer Design: Conventional vs. Machine Learning



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- Use function approximators and learn parameter configurations θ from data

[Shen and Lau, 2011], Fiber nonlinearity compensation using extreme learning machine for DSP-based ..., (*OECC*)
[Giacoumidis et al., 2015], Fiber nonlinearity-induced penalty reduction in CO-OFDM by ANN-based ..., (*Opt. Lett.*)
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- Joint transmitter–receiver learning via autoencoder [O'Shea and Hoydis, 2017]

[Karanov et al., 2018], End-to-end deep learning of optical fiber communications (*J. Lightw. Technol.*)
[Li et al., 2018], Achievable information rates for nonlinear fiber communication via end-to-end autoencoder learning, (*ECOC*)
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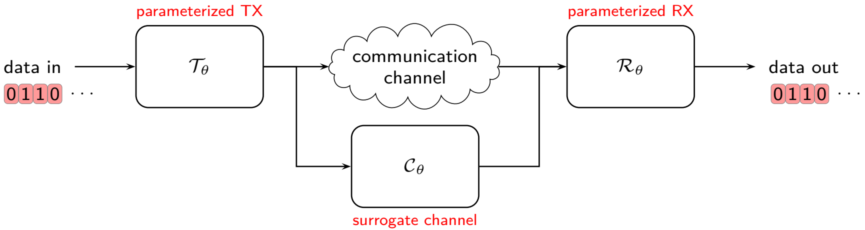

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- Use function approximators and learn parameter configurations θ from data
- Joint transmitter–receiver learning via autoencoder [O'Shea and Hoydis, 2017]
- Surrogate channel models for gradient-based TX training

[O'Shea et al., 2018], Approximating the void: Learning stochastic channel models from observation with variational GANs, (*arXiv*)
[Ye et al., 2018], Channel agnostic end-to-end learning based communication systems with conditional GAN, (*arXiv*)
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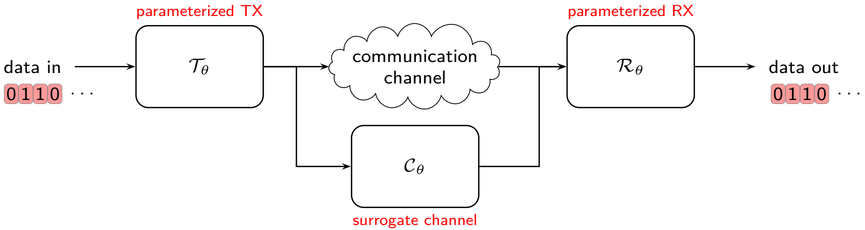

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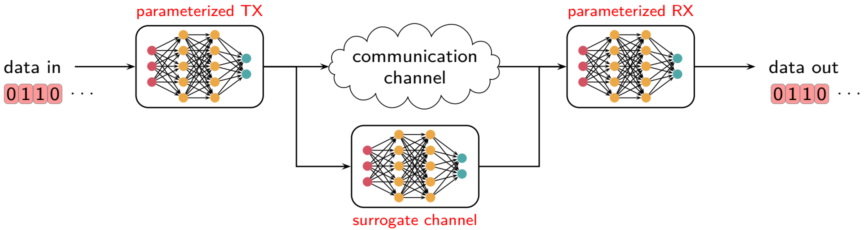
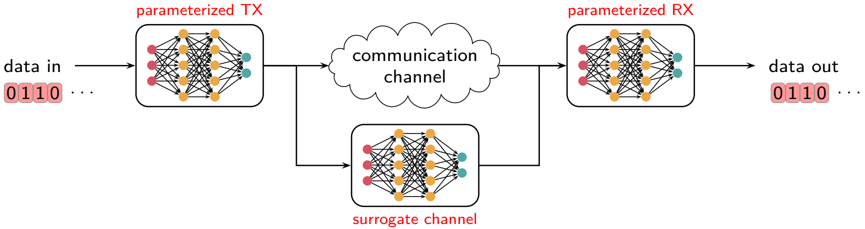


Using (deep) neural networks for $\mathcal{T}_\theta, \mathcal{R}_\theta, \mathcal{C}_\theta$? Possible, but ...

- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?
- ...

Physical-Layer Design: Conventional vs. Machine Learning



Using (deep) neural networks for $\mathcal{T}_\theta, \mathcal{R}_\theta, \mathcal{C}_\theta$? Possible, but ...

- How to choose the network architecture (#layers, activation function)? ✗
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Our contribution: designing "neural-network-like" machine-learning models by exploiting the underlying physics of the propagation.

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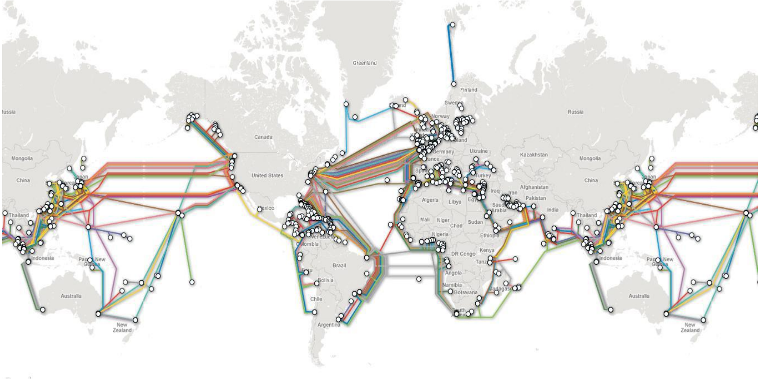
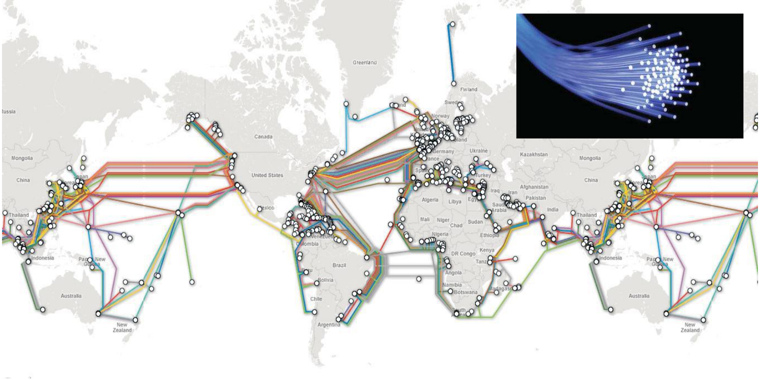
Fiber-Optic Communications



Fiber-optic systems enable data traffic over very long distances connecting cities, countries, and continents.

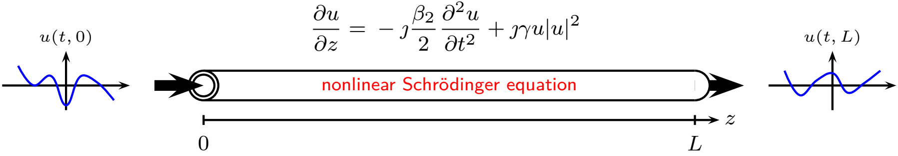
Fiber-Optic Communications



Fiber-optic systems enable data traffic over very long distances connecting cities, countries, and continents.

- Dispersion: different wavelengths travel at different speeds (linear)
- Kerr effect: refractive index changes with signal intensity (nonlinear)

Nonlinear Fiber Channel Modeling



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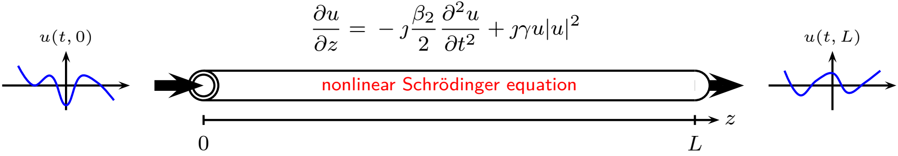

- Partial differential equation without general closed-form solution

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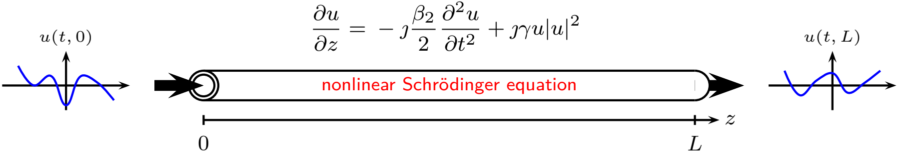
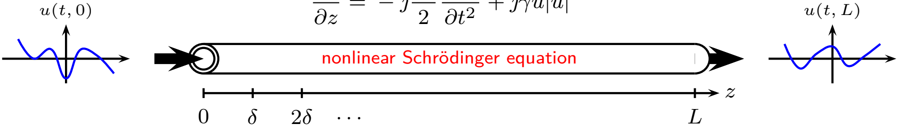


- Partial differential equation without general closed-form solution
- Sampling over a fixed time interval: $\boldsymbol{x} \in \mathbb{C}^n$ (input), $\boldsymbol{y} \in \mathbb{C}^n$ (output)

Nonlinear Fiber Channel Modeling

$$\frac{\partial u}{\partial z} = -\jmath \frac{\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \jmath \gamma u |u|^2$$



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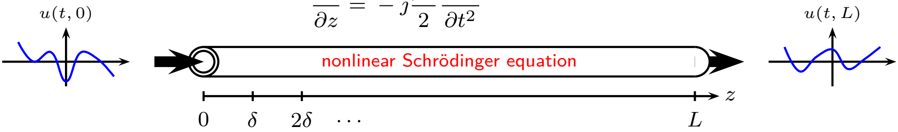

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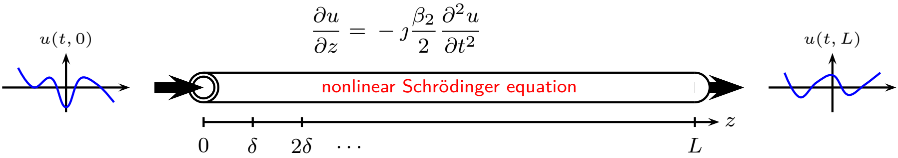

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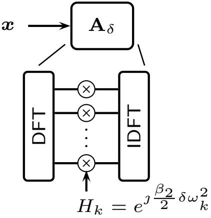

dispersion (all-pass filter)

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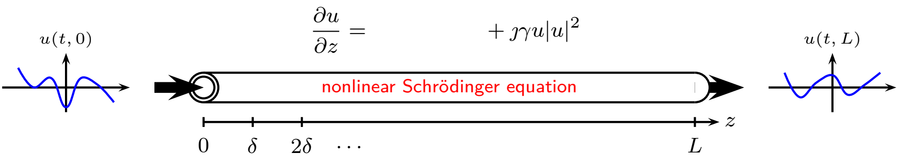

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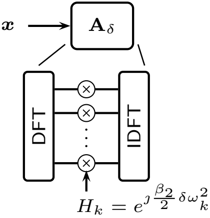

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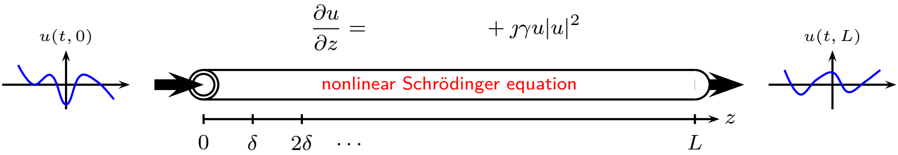

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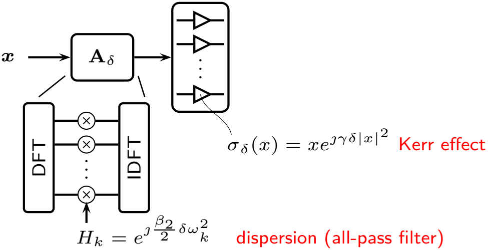

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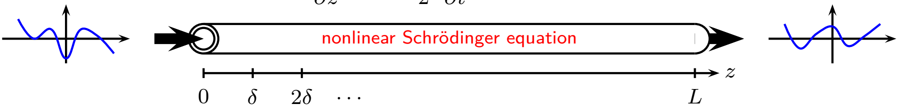

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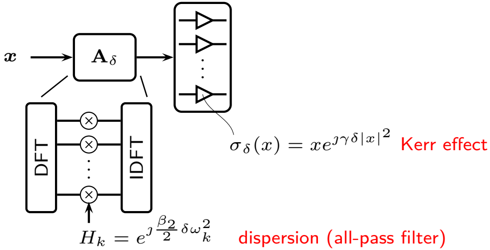

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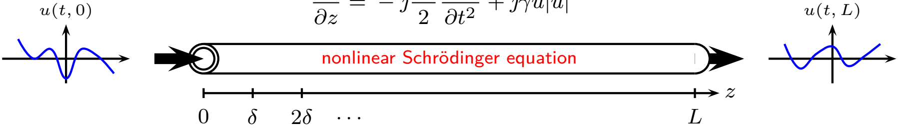

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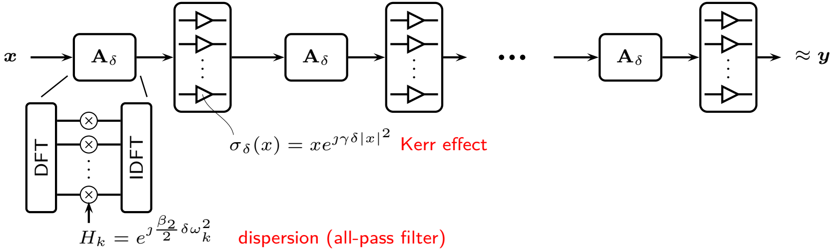

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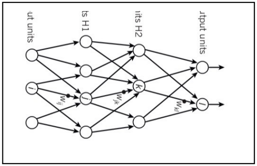

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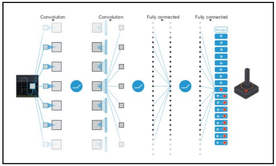

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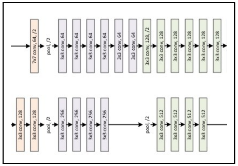

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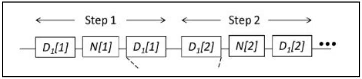
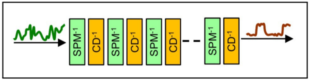
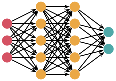


[Du and Lowery, 2010]

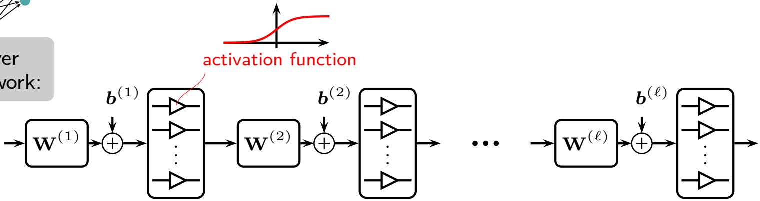


[Nakashima et al., 2017]

The Main Idea



multi-layer neural network:



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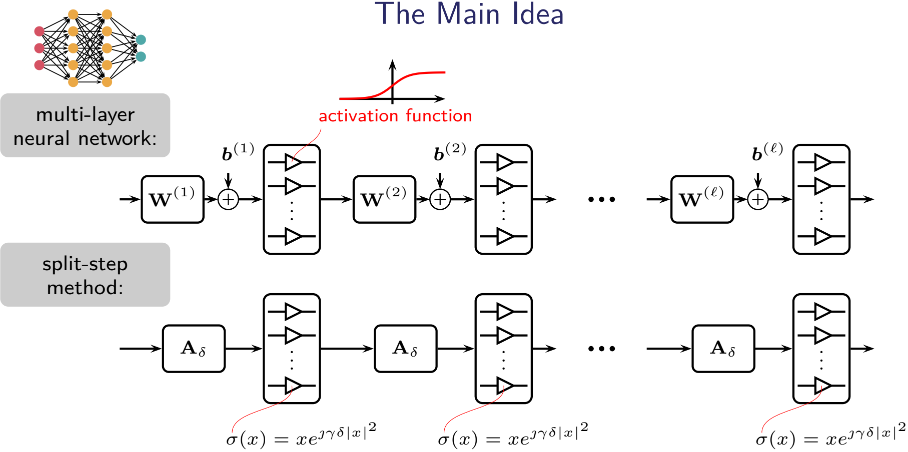

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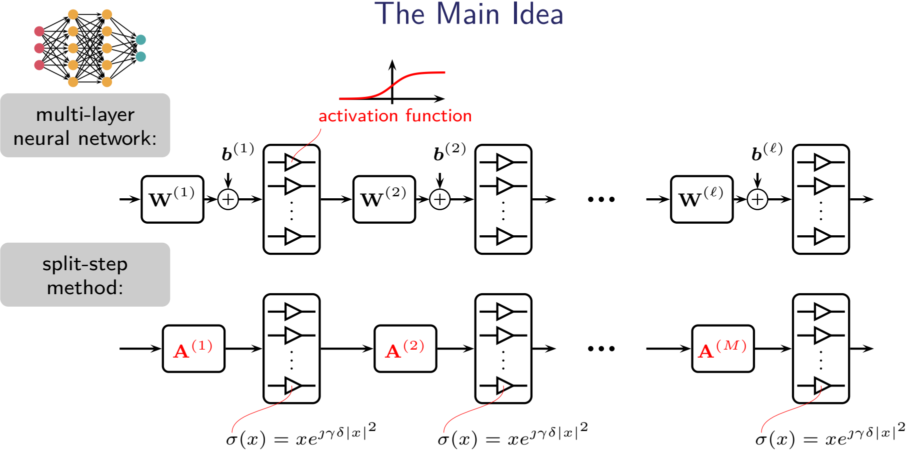

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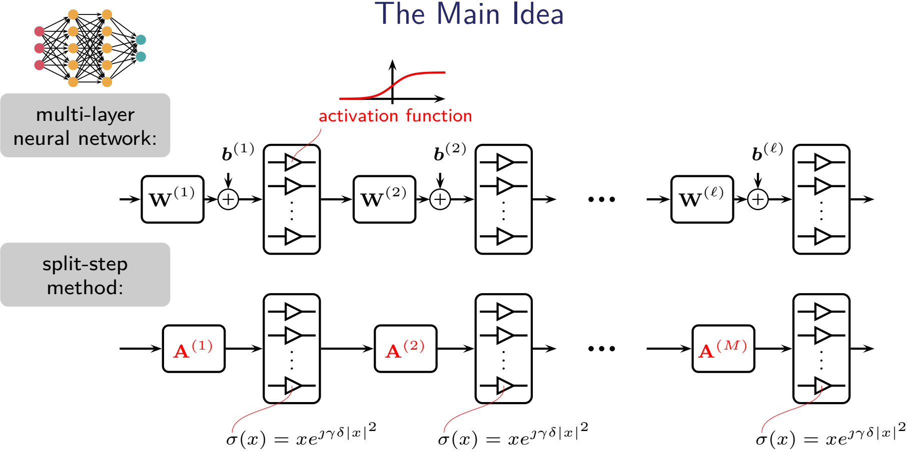
- Parameterize all linear steps: f_θ with $\theta = \{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(M)}\}$

[Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, *(OFC)*
[Häger & Pfister, 2021], Physics-Based Deep Learning for Fiber-Optic Communication Systems, *IEEE J. Sel. Areas Commun.*

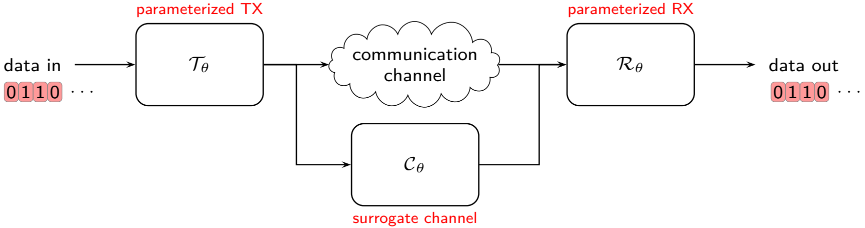
The Main Idea



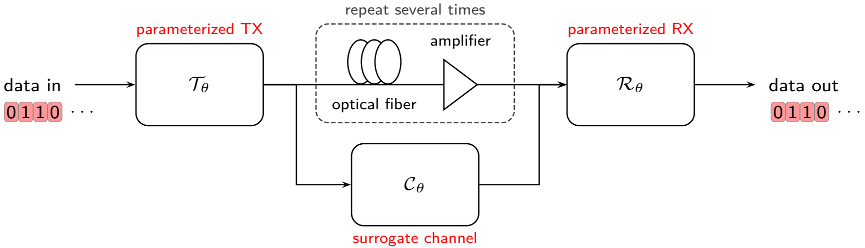
- This almost looks like a deep neural net!
- Parameterize all linear steps: f_θ with $\theta = \{\mathbf{A}^{(1)}, \ldots, \mathbf{A}^{(M)}\}$
- Special cases: step-size optimization, nonlinear operator "placement", . . .

[Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, *(OFC)*
[Häger & Pfister, 2021], Physics-Based Deep Learning for Fiber-Optic Communication Systems, *IEEE J. Sel. Areas Commun.*

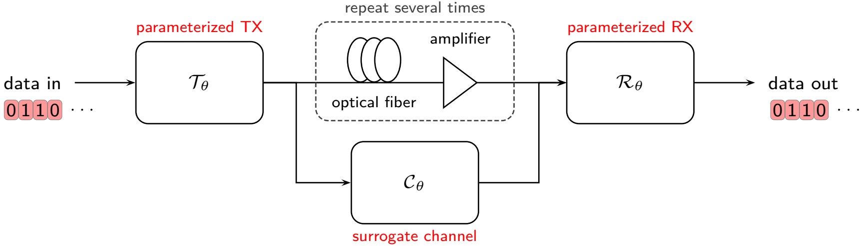
Possible Applications



Possible Applications

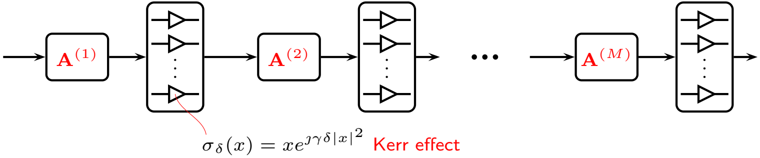


Possible Applications



- Channel $\mathcal{C}_\theta$: fine-tune model based on experimental data, reduce simulation time [Leibrich and Rosenkranz, 2003], [Li et al., 2005]
- Receiver $\mathcal{R}_\theta$: nonlinear equalization (focus in this talk)
- Transmitter $\mathcal{T}_\theta$: digital pre-distortion [Essiambre and Winzer, 2005], [Roberts et al., 2006], "split" nonlinearity compensation [Lavery et al., 2016]

Potential Benefits



- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?

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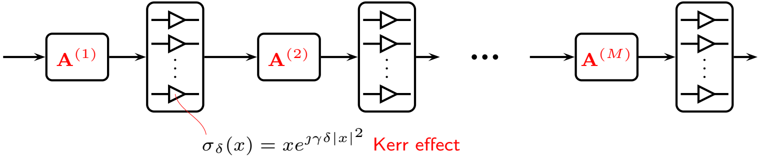

- How to choose the network architecture (#layers, activation function)? ✓
  - Activation function is fixed; number of layers = number of steps
  - Hidden feature representations ≈ signal at intermediate fiber locations
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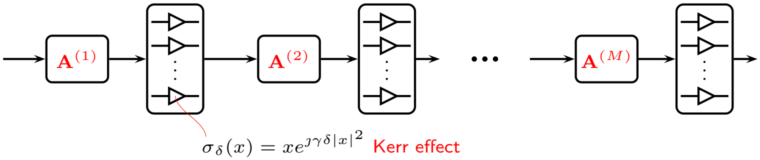

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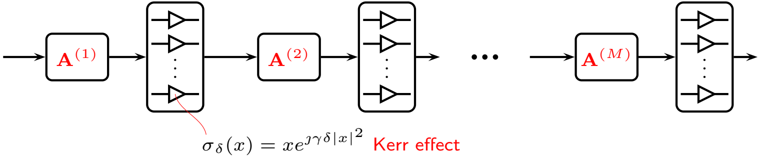

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  - Filter symmetry can be enforced in the linear steps
  - Model compression (e.g., parameter pruning, quantization)
- How to interpret the solutions? Any insight gained? ✓
  - Learned parameter configurations are interpretable
  - Satisfactory explanations for benefits over previous handcrafted solutions

Outline

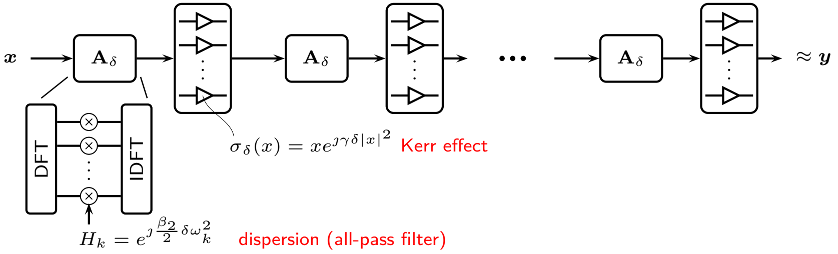
1. Machine Learning and Neural Networks for Communications

2. Physics-Based Machine Learning for Fiber-Optic Communications

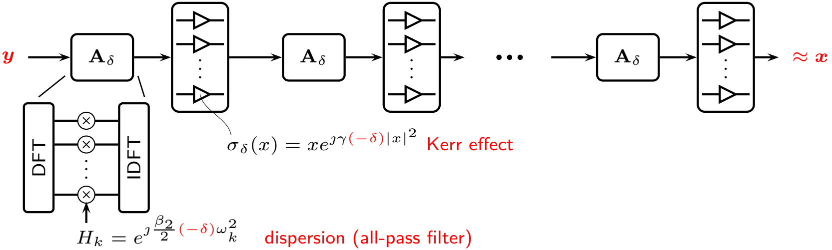
3. Learned Digital Backpropagation

4. Conclusions

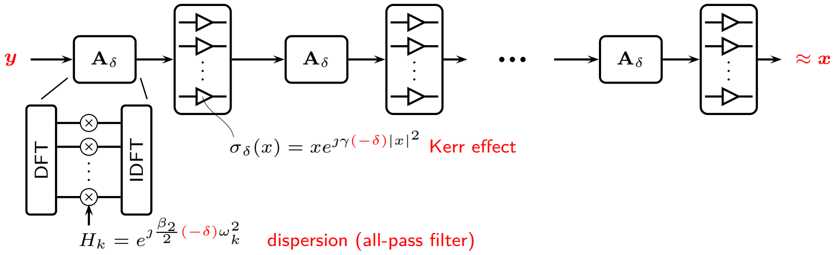
Digital Backpropagation



Digital Backpropagation

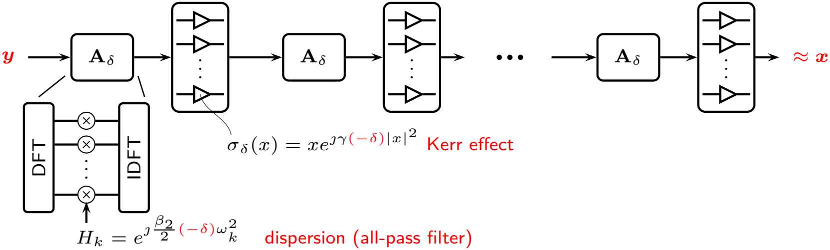


Digital Backpropagation



- Fiber with negated parameters ($\beta_2 \to -\beta_2$, $\gamma \to -\gamma$) would perform perfect channel inversion [Paré et al., 1996] (ignoring attenuation)

Digital Backpropagation



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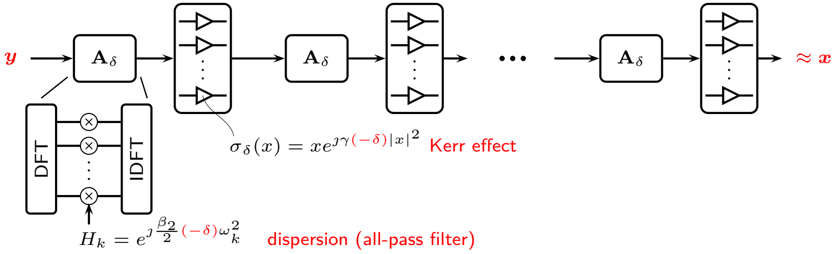
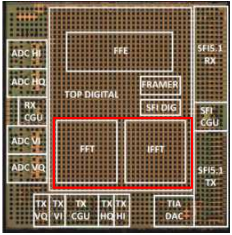
- Digital backpropagation: invert a partial differential equation in real time
  [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008]

Digital Backpropagation

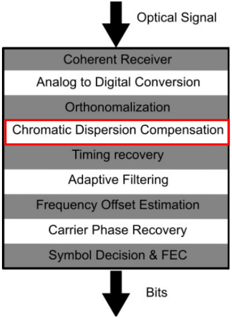


- Fiber with negated parameters ($\beta_2 \to -\beta_2$, $\gamma \to -\gamma$) would perform perfect channel inversion [Paré et al., 1996] (ignoring attenuation)
- Digital backpropagation: invert a partial differential equation in real time [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008]
- Widely considered to be impractical (too complex): linear equalization is already one of the most power-hungry DSP blocks in coherent receivers

Real-Time Digital Backpropagation



[Crivelli et al., 2014]



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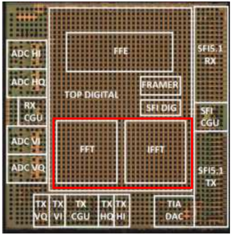

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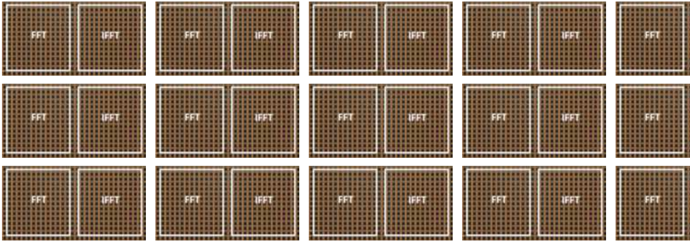

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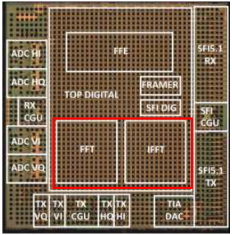

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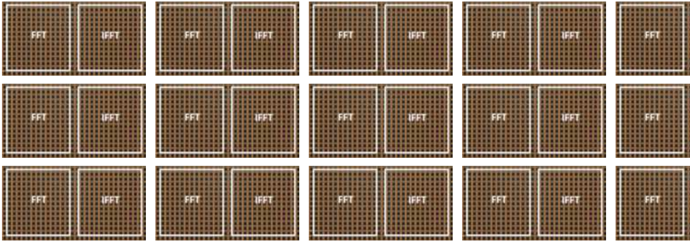

- Complexity increases with the number of steps M $\implies$ reduce M as much as possible (see, e.g., [Du and Lowery, 2010], [Rafique et al., 2011], [Li et al., 2011], [Yan et al., 2011], [Napoli et al., 2014], [Secondini et al., 2016], . . .)

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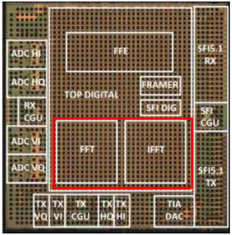

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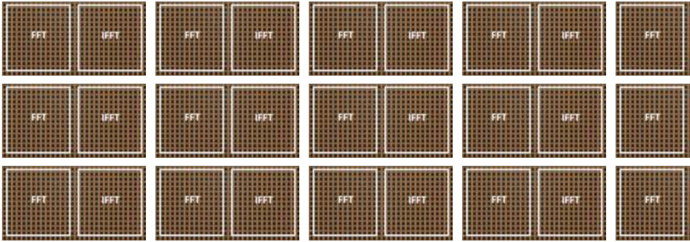

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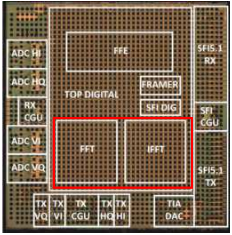

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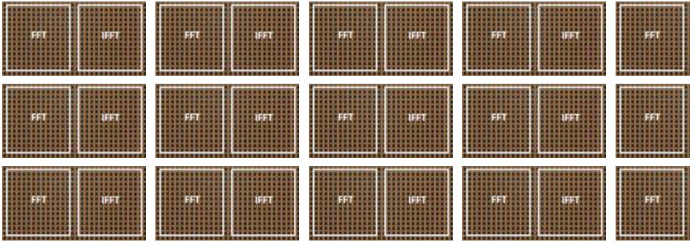

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- Intuitive, but . . . this flattens a deep (multi-layer) computation graph

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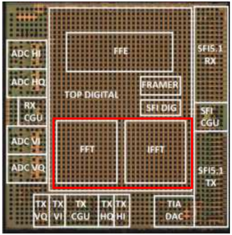

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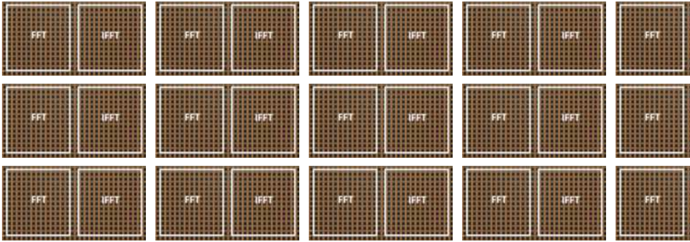

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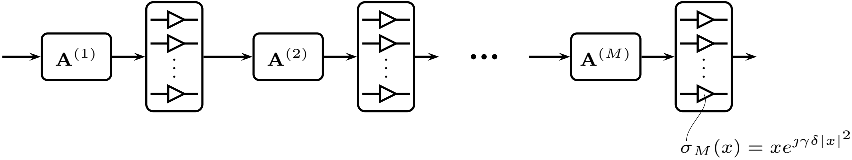
- Intuitive, but ... this flattens a deep (multi-layer) computation graph

Our approach: many steps but model compression

Joint optimization, pruning, and quantization of all linear steps $\implies$ hardware-efficient digital backpropagation

Learned Digital Backpropagation

TensorFlow implementation of the computation graph $f_\theta(\boldsymbol{y})$:



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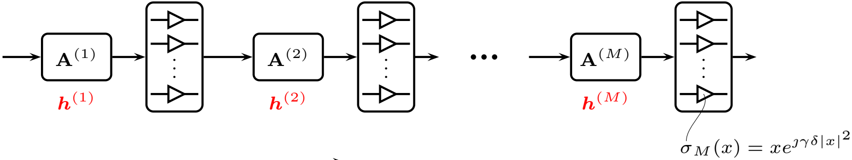
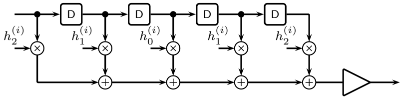


$$\begin{matrix}
h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & 0 & 0 & 0 & 0 & 0 & \\
h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & 0 & 0 & 0 & 0 & \\
h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & 0 & 0 & 0 & \cdots \\
h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & 0 & 0 & \\
h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & 0 & \\
0 & h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & h_4^{(i)} & \\
0 & 0 & h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & h_3^{(i)} & \\
0 & 0 & 0 & h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & h_2^{(i)} & \\
0 & 0 & 0 & 0 & h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & h_1^{(i)} & \\
0 & 0 & 0 & 0 & 0 & h_4^{(i)} & h_3^{(i)} & h_2^{(i)} & h_1^{(i)} & h_0^{(i)} & \\
 & & \vdots & & & & & & & & \ddots
\end{matrix}$$



finite impulse response (FIR) filter
complex & symmetric coefficients

Learned Digital Backpropagation

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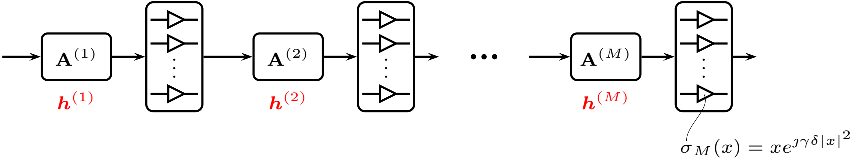

Deep learning of all FIR filter coefficients $\theta = \{\boldsymbol{h}^{(1)}, \dots, \boldsymbol{h}^{(M)}\}$:

$$\min_\theta \sum_{i=1}^{N} \text{Loss}(f_\theta(\boldsymbol{y}^{(i)}), \boldsymbol{x}^{(i)}) \triangleq g(\theta) \qquad \text{using} \quad \theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k)$$

mean squared error

Adam optimizer, fixed learning rate

Learned Digital Backpropagation

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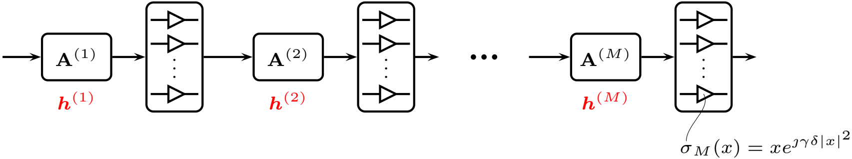

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mean squared error — Adam optimizer, fixed learning rate

Iteratively prune (set to 0) outermost filter taps during gradient descent

Iterative Filter Tap Pruning

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} \\ \boldsymbol{h}^{(2)} \\ \vdots \\ \boldsymbol{h}^{(M)} \end{cases}$$

Iterative Filter Tap Pruning

$\longleftarrow$ starting length $2K' + 1$ $\longrightarrow$

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} = (\; h_{K'}^{(1)} \; \cdots \; h_{K}^{(1)} \; \cdots \; h_{1}^{(1)} \; h_{0}^{(1)} \; h_{1}^{(1)} \; \cdots \; h_{K}^{(1)} \; \cdots \; h_{K'}^{(1)} \;) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\; h_{K'}^{(2)} \; \cdots \; h_{K}^{(2)} \; \cdots \; h_{1}^{(2)} \; h_{0}^{(2)} \; h_{1}^{(2)} \; \cdots \; h_{K}^{(2)} \; \cdots \; h_{K'}^{(2)} \;) & \text{step } 2 \\ \vdots \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots & \\ \boldsymbol{h}^{(M)} = (\; h_{K'}^{(M)} \; \cdots \; h_{K}^{(M)} \; \cdots \; h_{1}^{(M)} \; h_{0}^{(M)} \; h_{1}^{(M)} \; \cdots \; h_{K}^{(M)} \; \cdots \; h_{K'}^{(M)} \;) & \text{step } M \end{cases}$$

Iterative Filter Tap Pruning

$$\xleftarrow{\hspace{3cm}} \text{starting length } 2K'+1 \xrightarrow{\hspace{3cm}}$$

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} = (\; h^{(1)}_{K'} \; \cdots \; h^{(1)}_{K} \; \cdots \; h^{(1)}_{1} \; h^{(1)}_{0} \; h^{(1)}_{1} \; \cdots \; h^{(1)}_{K} \; \cdots \; h^{(1)}_{K'} \;) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\; h^{(2)}_{K'} \; \cdots \; h^{(2)}_{K} \; \cdots \; h^{(2)}_{1} \; h^{(2)}_{0} \; h^{(2)}_{1} \; \cdots \; h^{(2)}_{K} \; \cdots \; h^{(2)}_{K'} \;) & \text{step } 2 \\ \vdots \qquad\quad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots & \\ \boldsymbol{h}^{(M)} = (\; h^{(M)}_{K'} \; \cdots \; h^{(M)}_{K} \; \cdots \; h^{(M)}_{1} \; h^{(M)}_{0} \; h^{(M)}_{1} \; \cdots \; h^{(M)}_{K} \; \cdots \; h^{(M)}_{K'} \;) & \text{step } M \end{cases}$$

- Initially: constrained least-squares coefficients (LS-CO) [Sheikh et al., 2016]

Iterative Filter Tap Pruning

$$\overleftarrow{\longrightarrow}\ \text{starting length } 2K'+1$$

$$\overleftarrow{\longrightarrow}\ \text{target length } 2K+1$$

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} = (\ h_{K'}^{(1)} \ \cdots \ h_{K}^{(1)} \ \cdots \ h_{1}^{(1)} \ h_{0}^{(1)} \ h_{1}^{(1)} \ \cdots \ h_{K}^{(1)} \ \cdots \ h_{K'}^{(1)} \ ) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\ h_{K'}^{(2)} \ \cdots \ h_{K}^{(2)} \ \cdots \ h_{1}^{(2)} \ h_{0}^{(2)} \ h_{1}^{(2)} \ \cdots \ h_{K}^{(2)} \ \cdots \ h_{K'}^{(2)} \ ) & \text{step } 2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots & \\ \boldsymbol{h}^{(M)} = (\ h_{K'}^{(M)} \ \cdots \ h_{K}^{(M)} \ \cdots \ h_{1}^{(M)} \ h_{0}^{(M)} \ h_{1}^{(M)} \ \cdots \ h_{K}^{(M)} \ \cdots \ h_{K'}^{(M)} \ ) & \text{step } M \end{cases}$$

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Iterative Filter Tap Pruning

$$\longleftarrow \text{starting length } 2K'+1 \longrightarrow$$

$$\longleftarrow \text{target length } 2K+1 \longrightarrow$$

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} = (\; \cancel{h_{K'}^{(1)}} \; \cdots \; h_K^{(1)} \; \cdots \; h_1^{(1)} \; h_0^{(1)} \; h_1^{(1)} \; \cdots \; h_K^{(1)} \; \cdots \; \cancel{h_{K'}^{(1)}} \;) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\; h_{K'}^{(2)} \; \cdots \; h_K^{(2)} \; \cdots \; h_1^{(2)} \; h_0^{(2)} \; h_1^{(2)} \; \cdots \; h_K^{(2)} \; \cdots \; h_{K'}^{(2)} \;) & \text{step } 2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots & \\ \boldsymbol{h}^{(M)} = (\; h_{K'}^{(M)} \; \cdots \; h_K^{(M)} \; \cdots \; h_1^{(M)} \; h_0^{(M)} \; h_1^{(M)} \; \cdots \; h_K^{(M)} \; \cdots \; h_{K'}^{(M)} \;) & \text{step } M \end{cases}$$

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Iterative Filter Tap Pruning

starting length $2K' + 1$

target length $2K + 1$

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} = (\cancel{h^{(1)}_{K'}} \; \cdots \; h^{(1)}_{K} \; \cdots \; h^{(1)}_{1} \; h^{(1)}_{0} \; h^{(1)}_{1} \; \cdots \; h^{(1)}_{K} \; \cdots \; \cancel{h^{(1)}_{K'}}) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\cancel{h^{(2)}_{K'}} \; \cdots \; h^{(2)}_{K} \; \cdots \; h^{(2)}_{1} \; h^{(2)}_{0} \; h^{(2)}_{1} \; \cdots \; h^{(2)}_{K} \; \cdots \; \cancel{h^{(2)}_{K'}}) & \text{step } 2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots & \\ \boldsymbol{h}^{(M)} = (h^{(M)}_{K'} \; \cdots \; h^{(M)}_{K} \; \cdots \; h^{(M)}_{1} \; h^{(M)}_{0} \; h^{(M)}_{1} \; \cdots \; h^{(M)}_{K} \; \cdots \; h^{(M)}_{K'}) & \text{step } M \end{cases}$$

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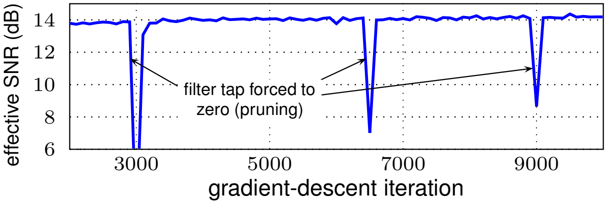
Iterative Filter Tap Pruning

starting length $2K'+1$

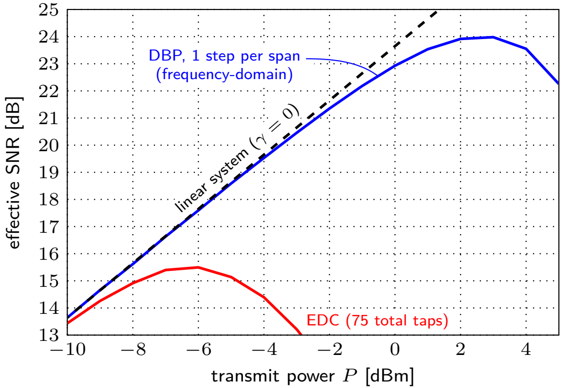
target length $2K+1$

$$
\theta = \begin{cases}
\boldsymbol{h}^{(1)} = (\; \cancel{h_{K'}^{(1)}} \; \cdots \; h_K^{(1)} \; \cdots \; h_1^{(1)} \; h_0^{(1)} \; h_1^{(1)} \; \cdots \; h_K^{(1)} \; \cdots \; \cancel{h_{K'}^{(1)}} \;) & \text{step } 1 \\
\boldsymbol{h}^{(2)} = (\; \cancel{h_{K'}^{(2)}} \; \cdots \; h_K^{(2)} \; \cdots \; h_1^{(2)} \; h_0^{(2)} \; h_1^{(2)} \; \cdots \; h_K^{(2)} \; \cdots \; \cancel{h_{K'}^{(2)}} \;) & \text{step } 2 \\
\vdots \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots & \\
\boldsymbol{h}^{(M)} = (\; h_{K'}^{(M)} \; \cdots \; h_K^{(M)} \; \cdots \; h_1^{(M)} \; h_0^{(M)} \; h_1^{(M)} \; \cdots \; h_K^{(M)} \; \cdots \; h_{K'}^{(M)} \;) & \text{step } M
\end{cases}
$$

- Initially: constrained least-squares coefficients (LS-CO) [Sheikh et al., 2016]
- Typical learning curve:



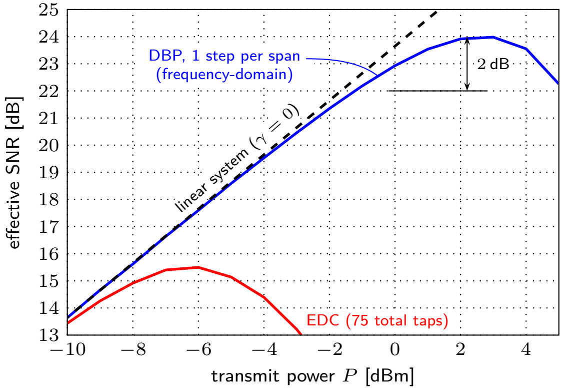
Revisiting Ip and Kahn (2008)



Parameters similar to [Ip and Kahn, 2008]:

- 25×80 km SSFM
- Gaussian modulation
- RRC pulses (0.1 roll-off)
- 10.7 Gbaud
- 2 samples/symbol processing
- single channel, single pol.

Revisiting Ip and Kahn (2008)



Parameters similar to [Ip and Kahn, 2008]:

- 25×80 km SSFM
- Gaussian modulation
- RRC pulses (0.1 roll-off)
- 10.7 Gbaud
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- $\gg 1000$ total taps (70 taps/step) $\implies$ $> 100\times$ complexity of EDC

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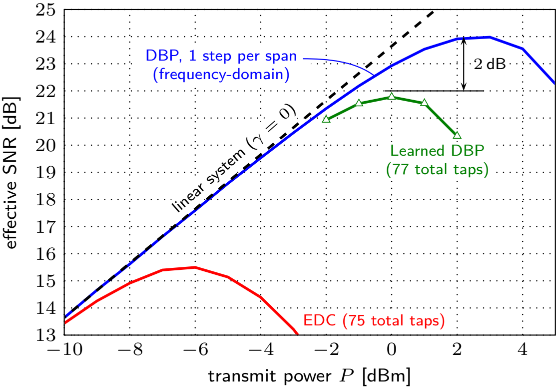

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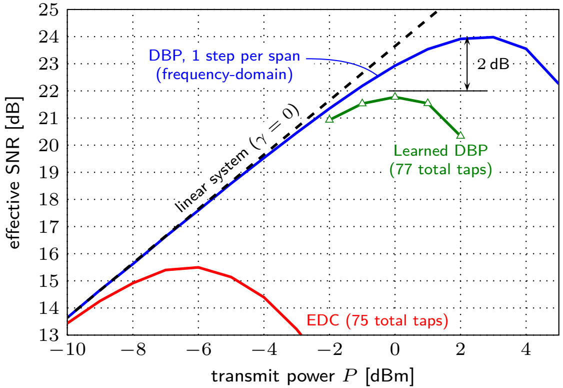

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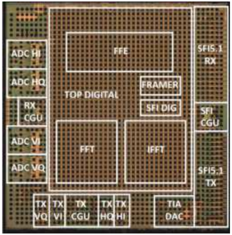
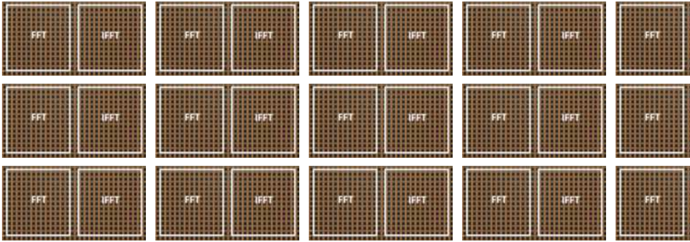
- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]
- Can outperform "ideal DBP" in the nonlinear regime [Häger and Pfister, 2018b]

Real-Time ASIC Implementation



[Crivelli et al., 2014]



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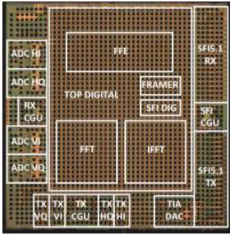

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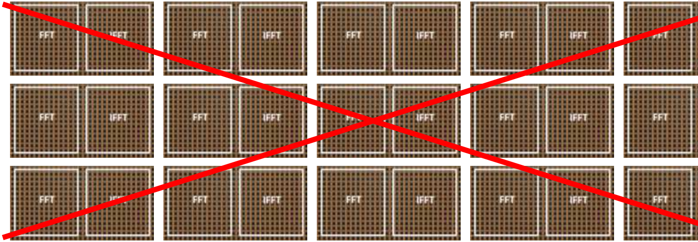

[Fougstedt et al., 2017], Time-domain digital back propagation: Algorithm and finite-precision implementation aspects, (*OFC*)
[Fougstedt et al., 2018], ASIC implementation of time-domain digital back propagation for coherent receivers, *(PTL)*
[Sherborne et al., 2018], On the impact of fixed point hardware for optical fiber nonlinearity compensation algorithms, (*JLT*)

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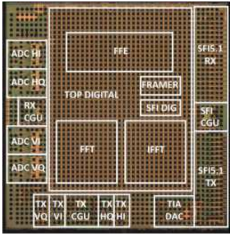

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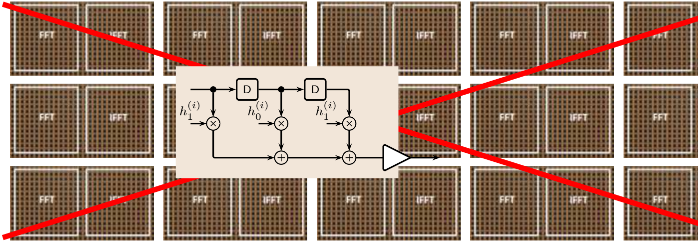

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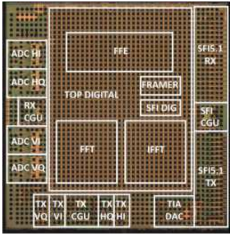

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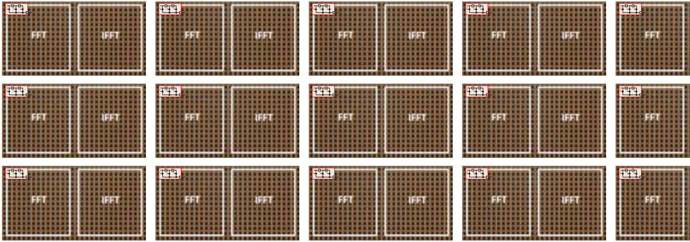

- Our linear steps are very short symmetric FIR filters (as few as 3 taps)
- 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
  - Only 5-6 bit filter coefficients via learned quantization
  - Hardware-friendly nonlinear steps (Taylor expansion)
  - All FIR filters are fully reconfigurable

[Fougstedt et al., 2018], ASIC implementation of time-domain digital backpropagation with deep-learned chromatic dispersion filters, *(ECOC)*

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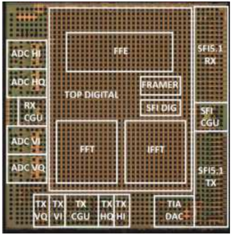

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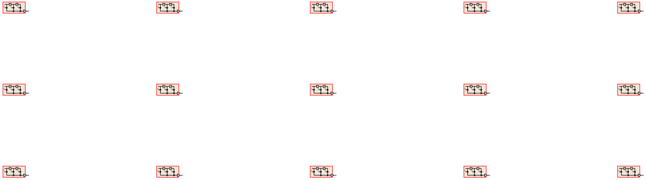

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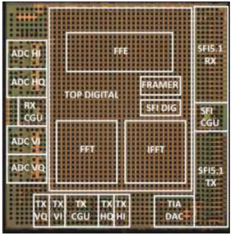

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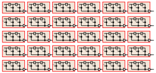

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- $< 2\times$ power compared to EDC [Crivelli et al., 2014, Pillai et al., 2014]

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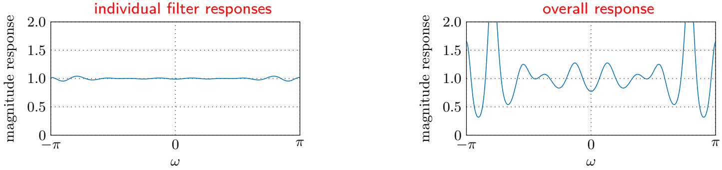
Why Does The Learning Approach Work?

Previous work: design a single filter or filter pair and use it repeatedly.

$\Longrightarrow$ Good overall response only possible with very long filters.



From [Ip and Kahn, 2009]:

- "We also note that [. . .] 70 taps, is much larger than expected"
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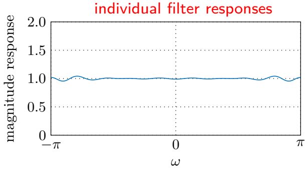

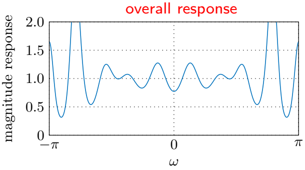

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The learning approach uncovered that there is no such requirement!

[Lian, Häger, Pfister, 2018], What can machine learning teach us about communications? (*ITW*)

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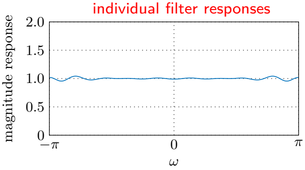
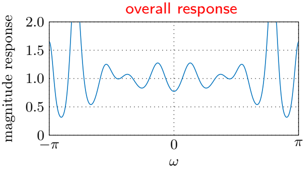
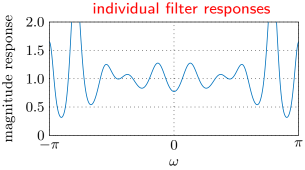
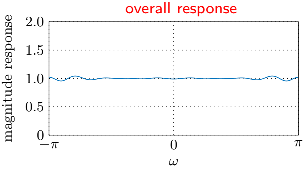




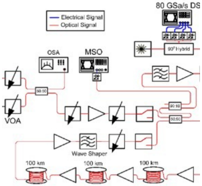
Sacrifice individual filter accuracy, but different response per step.

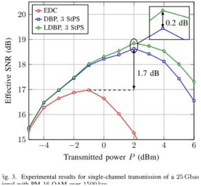
$\Longrightarrow$ Good overall response even with very short filters by joint optimization.

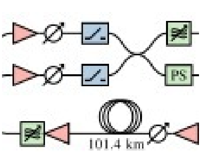


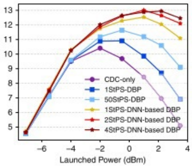


Experimental Investigations









Training with real-world data sets including presence of various hardware impairments (phase noise, timing error, frequency offset, etc.)

- [Oliari et al., 2020], Revisiting Efficient Multi-step Nonlinearity Compensation with Machine Learning: An Experimental Demonstration, *(J. Lightw. Technol.)*
- [Sillekens et al., 2020], Experimental Demonstration of Learned Time-domain Digital Back-propagation, *(Proc. IEEE Workshop on Signal Processing Systems)*
- [Fan et al., 2020], Advancing Theoretical Understanding and Practical Performance of Signal Processing for Nonlinear Optical Communications through Machine Learning, *(Nat. Commun.)*
- [Bitachon et al., 2020], Deep learning based Digital Back Propagation Demonstrating SNR gain at Low Complexity in a 1200 km Transmission Link, *(Opt. Express)*

Outline

1. Machine Learning and Neural Networks for Communications
2. Physics-Based Machine Learning for Fiber-Optic Communications
3. Learned Digital Backpropagation
4. Conclusions

The Bigger Picture

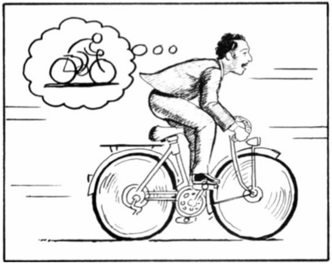
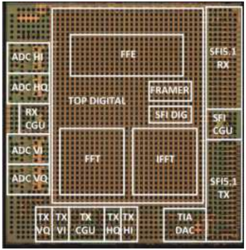


Figure 1. A World Model, from Scott McCloud's *Understanding Comics.* (McCloud, 1993; E, 2012)



[Crivelli et al., 2014]

- Optical receivers build models of their "environment"

[Ha & Schmidhuber, 2018], "World Models", arXiv:1803.10122 [cs.LG]

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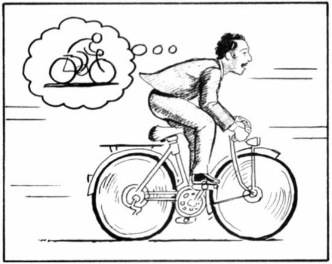

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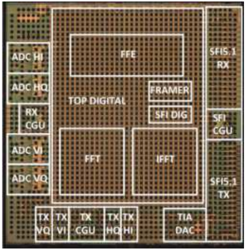

[Crivelli et al., 2014]

- Optical receivers build models of their "environment"
- Currently these models are linear and/or rigid (non-adaptive)
- Interpretable physics-based "multi-layer" models for machine learning can be obtained by exploiting our existing domain knowledge

[Ha & Schmidhuber, 2018], "World Models", arXiv:1803.10122 [cs.LG]

Conclusions

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neural-network-based ML
universal function approximators
good designs require experience and fine-tuning
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neural-network-based ML	**model-based ML**
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black boxes, difficult to “open”	familiar building blocks (e.g., FIR filters) can enable interpretability

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[Häger & Pfister, 2021], “Physics-Based Deep Learning for Fiber-Optic Communication Systems”, in *IEEE J. Sel. Areas Commun.*, see `https://arxiv.org/abs/2010.14258`

Code: `https://github.com/chaeger/LDBP`

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Thank you!

FORCE FIBER-OPTIC COMMUNICATIONS RESEARCH CENTER

Duke UNIVERSITY

TU/e

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