Karlsruhe Institute of Technology

On the Transmission Capacity of Wireless Multi-Channel Ad Hoc Networks with local FDMA scheduling

Jens P. Elsner, Ralph Tanbourgi, Friedrich K. Jondral

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Communications Engineering Lab
Prof. Dr.rer.nat. Friedrich K. Jondral

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Overview

On the Transmission Capacity of Wireless Multi-Channel Ad Hoc Networks with local FDMA scheduling

- Wireless ad hoc networks: Why multi-channel?
- Modeling using stochastic geometry
- Transmission Capacity with local FDMA scheduling
- On-going research
- Discussion / Q&A

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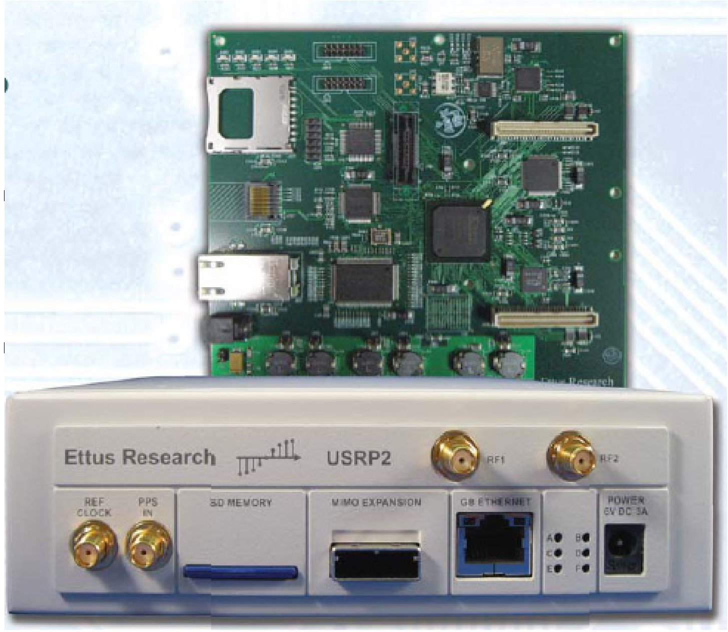
Wireless Multi-Channel Ad Hoc Networks: Motivation

Every nodes has limited RF bandwidth: multi-channel networks are necessary.

Target platform



e.g. USRP2

Requirements

- **Infrastructureless communication between arbitrary nodes**
- **High robustness against external interference**
- **High number of nodes**

- **Requirements call for multi-channel ad hoc networks with a flexible FDMA component**

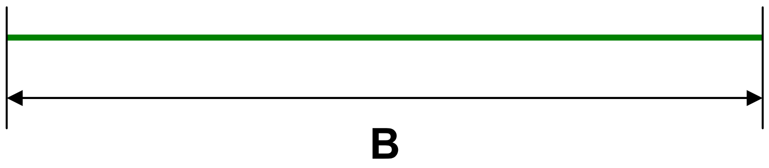
Source: Ettus Research LLC, http://www.ettus.com

Wireless Multi-Channel Ad Hoc Networks: Properties

Multi-channel ad hoc networks substantially differ from single channel networks.

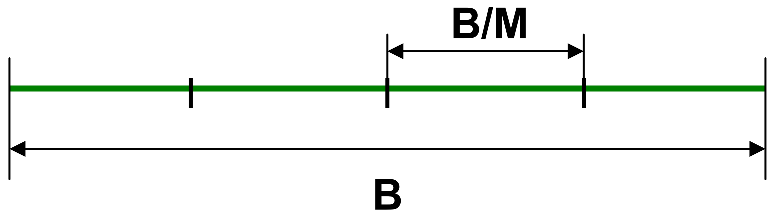
Single channel network

- PHY: More bandwidth is better
- MAC: CSMA/MACAW/IEEE 802.11 …
- Extensive research body available



Multi-channel network

- PHY: How to choose the bandwidth of a single channel?
- MAC: CSMA and derivatives not possible, different solutions needed
- Up until now low relevance in applications, less extensive research body



Multi-Channel Ad Hoc Networks: Research question

What can be gained by local FDMA in ad hoc networks?

Fundamental limits / system design

- What is the optimal split for the operation bandwidth / what is the optimum system bandwidth?
- Which number of nodes can be supported?
- What can be gained by scheduling in the communication range in ad hoc networks?

?

Wireless Multi-Channel Ad Hoc Networks: Modeling

Stochastic geometry offers a possibility to describe ad hoc networks analytically.

System model

- Node positions of interfering transmitters are described by a homogeneous Poisson point process (PPP).
- The PPP model offers analytical tractability and creates a homogeneous interference field.
- Metric is Shannon outage capacity; Receiver is assumed to work above an SINR threshold; Interference is AWGN
- Reference connection (cf. Slivnyak‘s Theorem) describes the whole network.

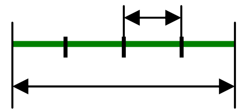
Possible statements

- Influence of parameters such as bandwidth, path loss exponent, node density, transmission range …
- Comparison of protocol strategies such as DSSS-CDMA, FH-CDMA, SIC, FDMA with scheduling etc.
- Model averages node positions, if a PPP is a realistic assumption has to be decided on case to case basis

S. Weber, J. Andrews, N. Jindal, *An overview of the transmission capacity of wireless networks,* submitted to IEEE Transactions on Communications, March 2010, under revision, arXiv:0809.0016v4

Wireless Multi-Channel Ad Hoc Networks: Modeling

Multi-channel model



SINR

$$\mathrm{SINR} = \frac{\rho r^{-\alpha}}{N_0 B_m + \sum_{i \in \Pi_m(\lambda_m)} \rho |X_i|^{-\alpha}}$$

Outage probability

$$q_m(\lambda_m) = \mathbb{P}\{B_m \log_2(1 + \mathrm{SINR}) \leq R_m\}$$

Transmission Capacity

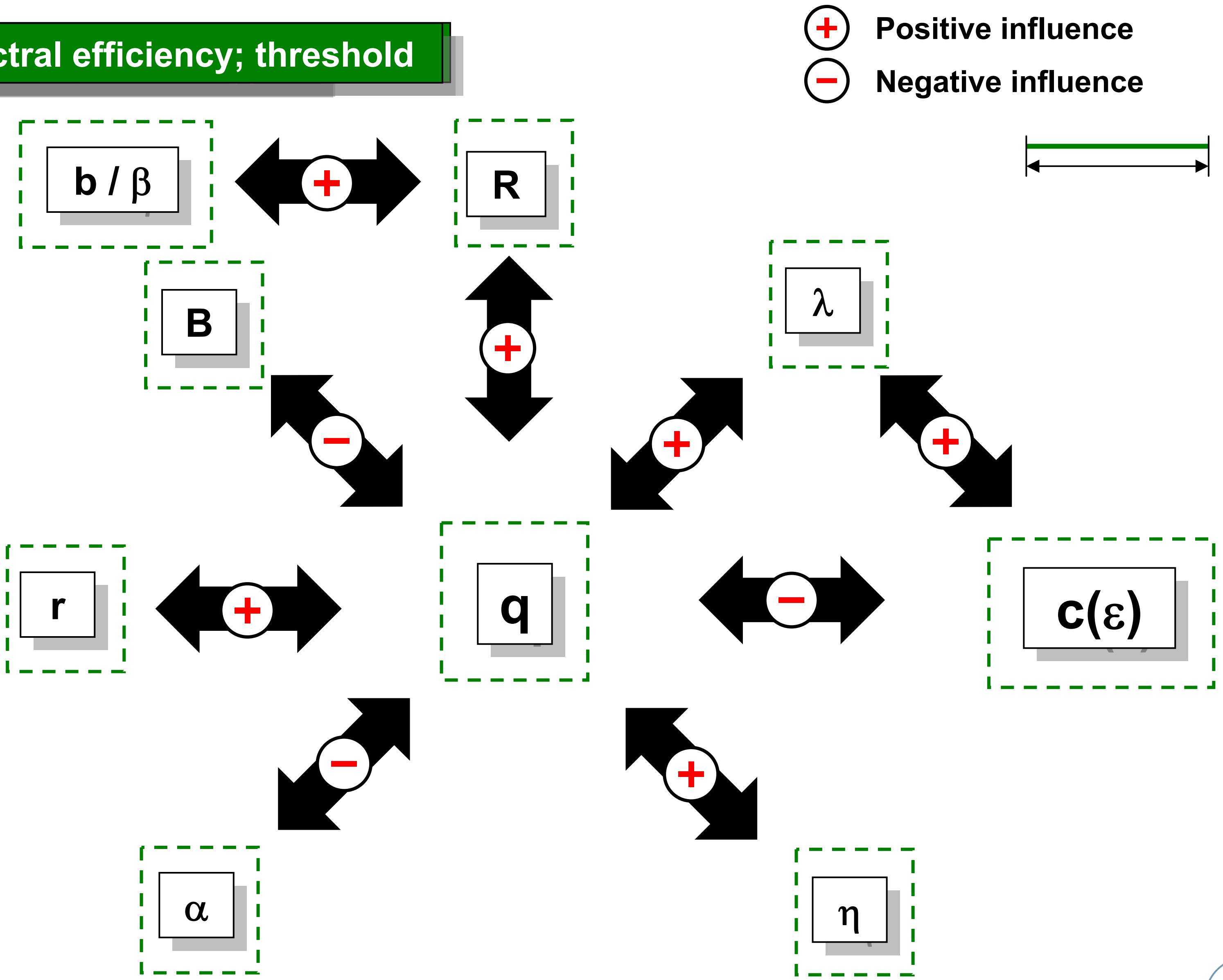
$$c_m(q_m) = \lambda_m(q_m)(1 - q_m)$$

$$c(\epsilon) = \sum_{m=1}^{M} c_m(\epsilon), \epsilon \in (0,1)$$

- ρ: transmission power
- r: communication distance
- α: path loss exponent
- $\eta_m = N_0 B_m$: noise
- λ_m: interferer density
- X_i: interferer positions
- q_m: outage probability
- $B_m = B/M$: bandwidth
- R_m: transmission rate
- c/c_m: Transmission Capacity

Influence of parameters for a single channel

b / β: spectral efficiency; threshold



Multi-channel model

Outage probability

$$q_m(\lambda_m) = \mathbb{P}\{B_m \log_2(1+\mathrm{SINR}) \leq R_m\}$$
$$= \mathbb{P}\{\underbrace{\mathrm{SINR}^{-1}}_{Y_m} > \underbrace{\frac{1}{2^{\frac{R_m}{B_m}} - 1}}_{=:1/\beta}\}$$

Map 2D-PPP to 1D-PPP

$$Y_m = \frac{N_0 B_m}{\rho r^{-\alpha}} + (\pi r^2 \lambda_m)^{\frac{\alpha}{2}} \underbrace{\sum_{i \in \Pi_1(1)} T_i^{-\frac{\alpha}{2}}}_{Z_\alpha}$$

Wireless Multi-Channel Ad Hoc Networks: Modeling

Multi-channel model

Outage probability

$$q_m(\lambda_m, R_m) = \mathbb{P}\{Z_\alpha > (\pi r^2 \lambda_m)^{-\frac{\alpha}{2}} \underbrace{(\frac{1}{2^{\frac{R_m}{B_m}} - 1} - \frac{N_0 B_m}{\rho r^{-\alpha}})}_{\theta_m}\}$$

$$q_m(\lambda_m) = \overline{F}_{Z_\alpha}((\pi r^2 \lambda_m)^{-\frac{\alpha}{2}} \theta_m)$$

Z_α

- In general no closed form solution for distribution of Z_α
- Exception: α=4

α = 4

$$c(\epsilon) = \frac{\sqrt{2/\pi}Q^{-1}((1+\epsilon)/2)}{\pi r^2 \sqrt{\beta}}(1-\epsilon)M$$

Wireless Multi-Channel Ad Hoc Networks: Modeling

Optimum split of operation bandwidth B.

Optimization problem

$$M_{\text{opt}} := \arg\min_{M} q_m(\lambda_m, R_m)$$

Wireless Multi-Channel Ad Hoc Networks: Modeling

Optimum split of operation bandwidth B.

Optimization problem

$$
\begin{aligned}
M_{\text{opt}} &:= \arg\min_{M} \left(\overline{F}_{Z_\alpha} \left(\frac{\theta_m}{(\pi r^2 \lambda_m)^{\frac{\alpha}{2}}} \right) \right) \\
&= \arg\max_{M} \left(\frac{\theta_m}{(\pi r^2 \lambda_m)^{\frac{\alpha}{2}}} \right) \\
&= \arg\max_{M} \left(\left((\pi r^2 \frac{\lambda}{M})^{-\frac{\alpha}{2}} \left(\frac{1}{2^{\frac{R_m}{B}M} - 1} - \frac{N_0 B}{\rho r^{-\alpha} M} \right) \right) \right) \\
&= \arg\max_{M} \left(M^{\frac{\alpha}{2}} \left(\frac{1}{2^{\frac{R_m}{B}M} - 1} - \underbrace{\frac{N_0 B}{\rho r^{-\alpha} M}}_{N_0/E_b} \right) \right)
\end{aligned}
$$

Wireless Multi-Channel Ad Hoc Networks: Modeling

In interference limited networks, optimum split of operation bandwidth B depends on α only.

Optimization problem

$$M_{\text{opt}} := \arg\min_{M} q_m(\lambda_m, R_m)$$

$\frac{E_b}{N_0} \to \infty$

$$M_{\text{opt}} = \frac{B}{R_m} \underbrace{\log_2 \exp\left(\frac{\alpha}{2} + \mathcal{W}\left(-\frac{\alpha}{2}\exp\left(-\frac{\alpha}{2}\right)\right)\right)}_{b_{\text{opt}}}$$

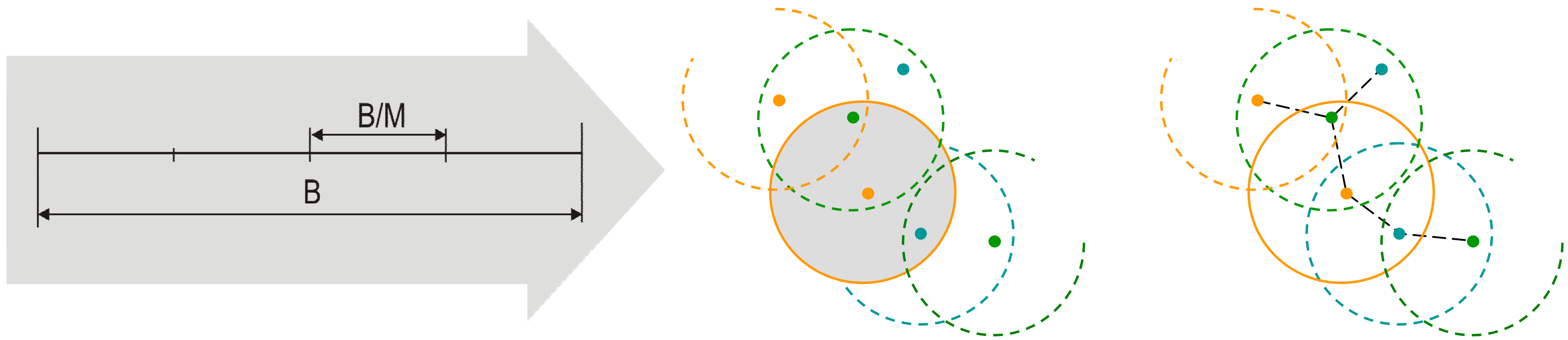
Solution

N. Jindal, S. Weber, J. Andrews, *Bandwidth partitioning for Ad Hoc networks*, IEEE Transactions on Communications, Aug 2008

Multi-Channel Ad Hoc Networks: Local FDMA

Neighbors in communication range r use different channels.

Local FDMA in ad hoc networks



Brooks' theorem

- Node coloring with M colors possible, if no node has more than M neighbors.

R. L. Brooks, *On colouring the nodes of a network*, Mathematical Proceedings of the Cambridge Philosophical Society, Apr 1941

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Multi-Channel Ad Hoc Networks: Local FDMA

When is locally orthogonal transmission possible on a network scale with high probability?

System model

- **Reference transmission (cf. Slivnyak's Theorem) describes the whole network**
  - **Condition: Local results apply globally.**
  - **Local results apply, if network orthogonalization is feasible with high probability.**
- Node positions are described by a homogeneous Poisson point process.
- **Network is limited to K nodes.**

Network orthogonalization

$$\mathrm{P}\left\{\max\{N_1, N_2, \ldots, N_K\} \leq M-1\right\} > 1-\epsilon_o$$

$$1-\epsilon_o(M) < \Big(\sum_{i=0}^{M-1} \exp(-\lambda_n)\frac{\lambda_n^i}{i!}\Big)^K$$

$$= \Phi(M, \lambda_n)^K$$

$$M \geq \Phi^{-1}\left((1-\epsilon_o)^{\frac{1}{K}}, \lambda_n\right)$$

$\lambda_n = \pi r^2 \lambda$: Mean number of neighbors

K. Briggs, L. Song, T. Prellberg, A note on the distribution of the maximum of a set of Poisson random variables, preprint, http://front.math.ucdavis.edu/0903.4373, March 2009

Multi-Channel Ad Hoc Networks: Local FDMA

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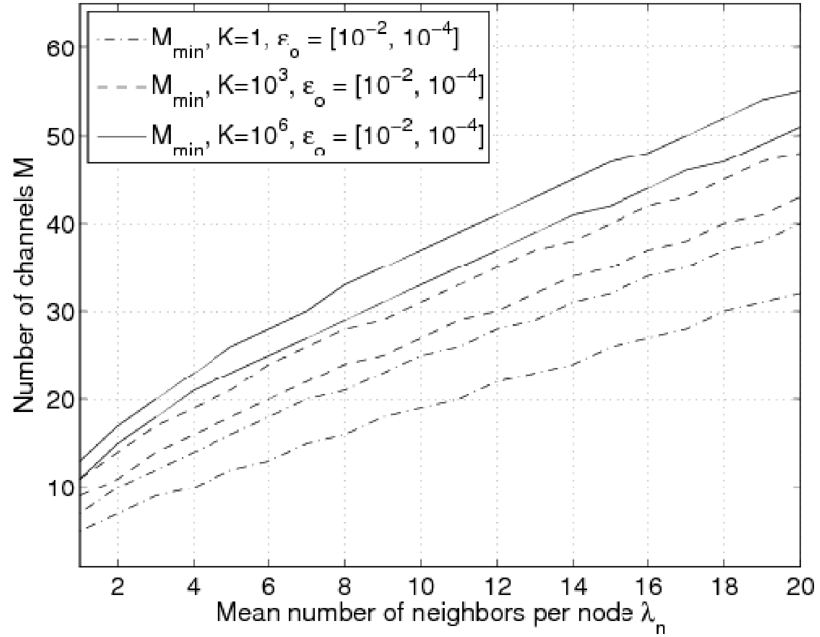
Network orthogonalization



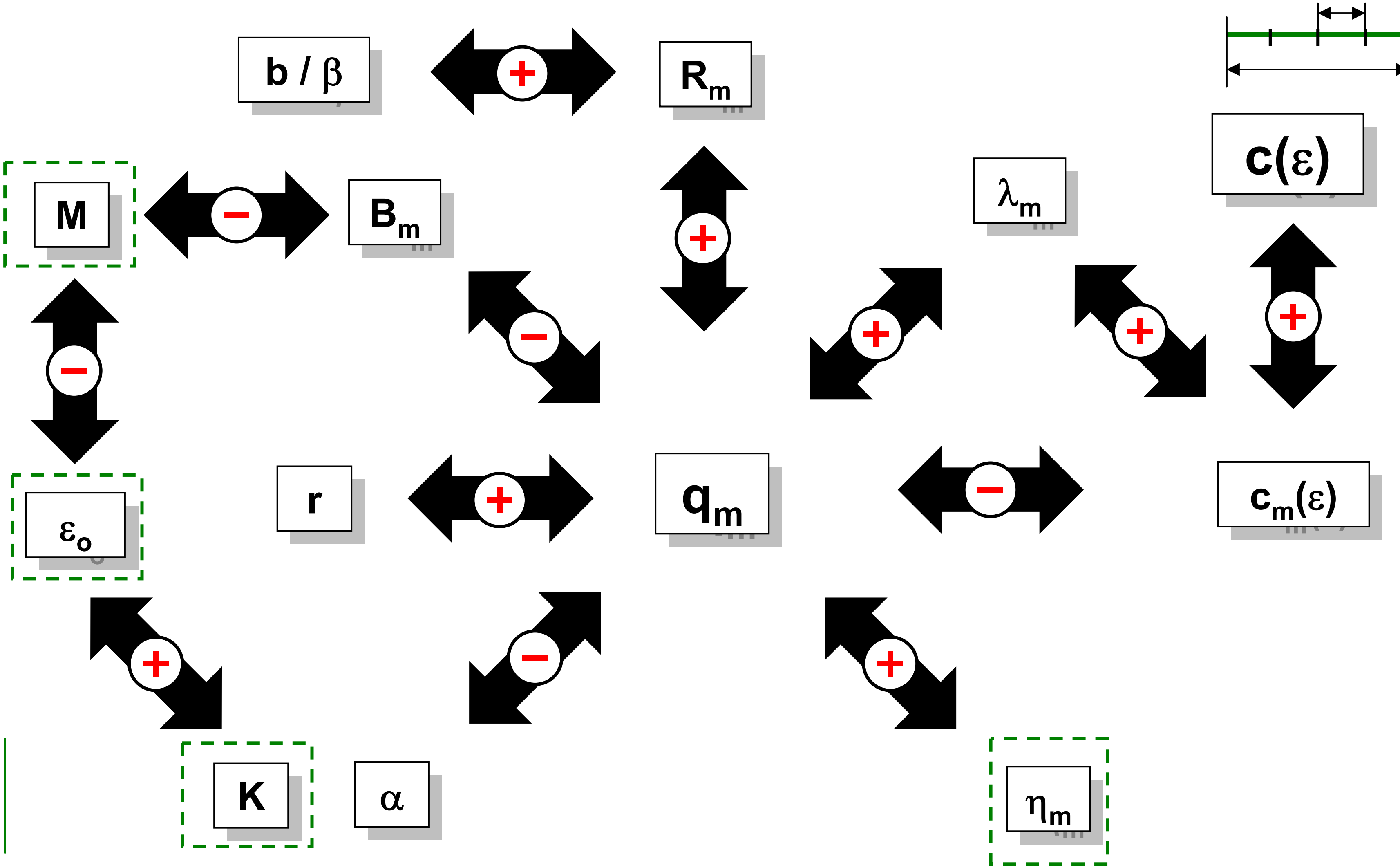
$$M \geq \Phi^{-1}\left((1-\epsilon_o)^{\frac{1}{K}}, \lambda_n\right)$$

K. Briggs, L. Song, T. Prellberg, A note on the distribution of the maximum of a set of Poisson random variables, preprint, http://front.math.ucdavis.edu/0903.4373, March 2009

Influence of parameters in multi-channel model

K: number of nodes in the network

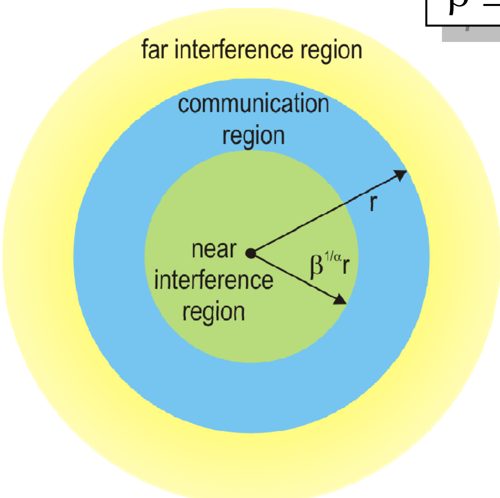
Positive influence
Negative influence



Local FDMA: Results

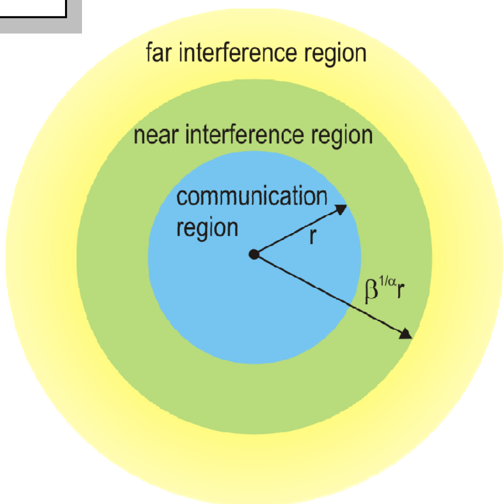
Local FDMA: No analytical solution for SIR distribution, lower and upper bounds needed.

$\beta \le 1$



$$p_o = \sum_{i=0}^{M-1} \exp(-\lambda_n)\frac{\lambda_n^i}{i!}$$

$\beta > 1$



$$1-q(\lambda) = p_o(\lambda)\mathbb{P}\{Y_r(\lambda)\le\beta^{-1}\} + (1-p_o(\lambda))\;\mathbb{P}\{\text{no node within } r_s\}\mathbb{P}\{Y_{r_s}(\lambda)\le\beta^{-1}\}$$

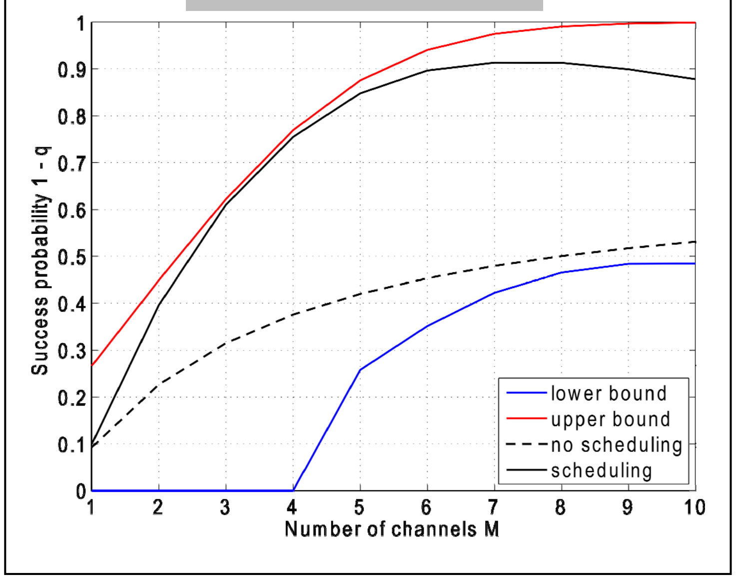
$$1-q(\lambda) = p_o(\lambda)\mathbb{P}\{\text{no node in annulus}\}\;\mathbb{P}\{Y_{r_s}(\lambda)\le\beta^{-1}\}$$

Local FDMA: Results

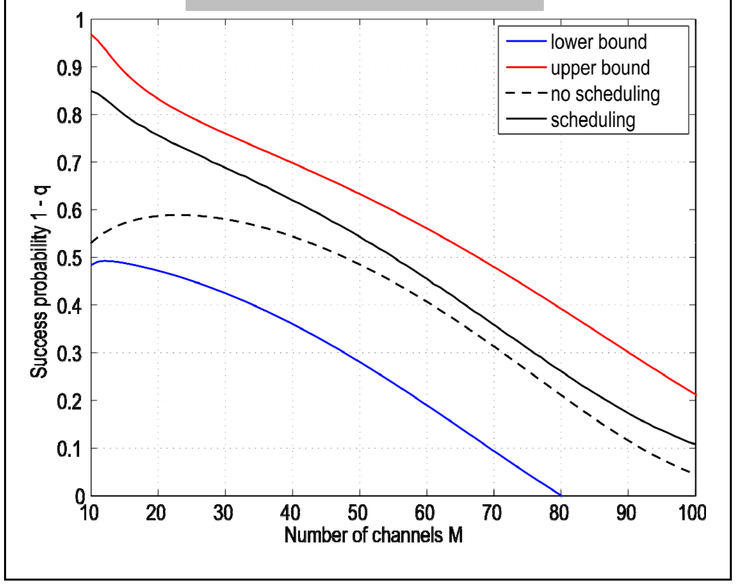
Local FDMA scheduling lowers outage probability.

Figures for $R_m/B = 0.1$, $\lambda_n = 5$, $\alpha = 4$ and $r = 10$

Success probability $\beta \leq 1$



Success probability $\beta > 1$

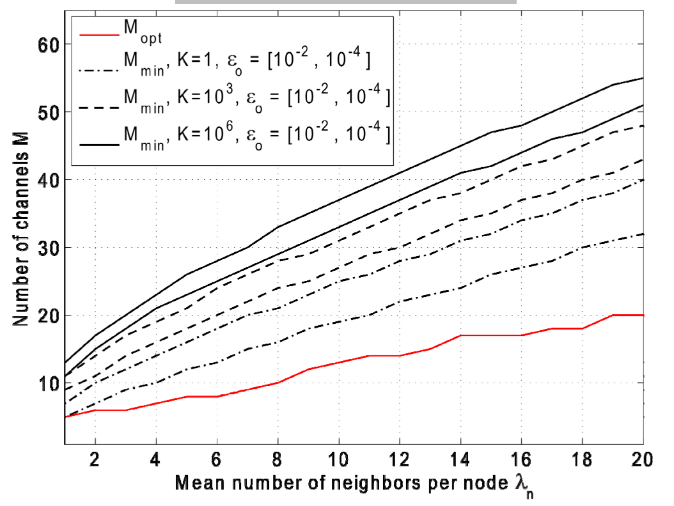


Local FDMA: Results

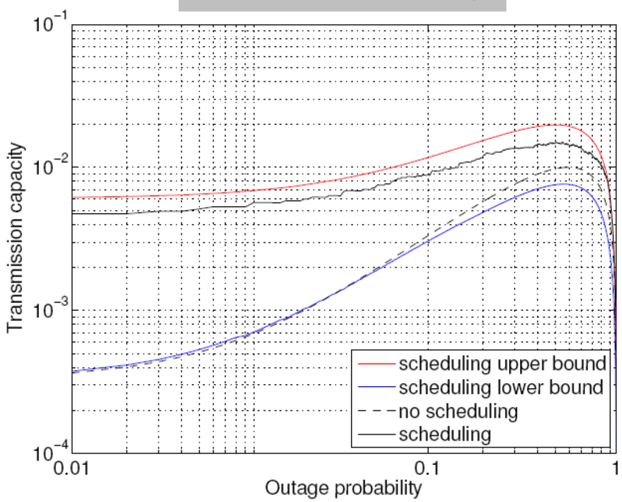
Local FDMA: Network orthogonalization dominates number of channels.

Figures for $R_m/B = 0.1,\ \lambda_n = 5,\ \alpha = 4$ **and** $r = 10$

Network orth. feasibility



Transmission capacity



Insights

- Optimum number of channels now depends on λ
- High gains possible (here: factor 1.35 - 13)

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Local FDMA: Comparison with SIC

FDMA vs. SIC: Local FDMA performs better, especially for low node densities.

Successive Interference Cancelation

- **SIC model: Interference power within r is mitigated by a factor 1-z, describing the cancelation efficiency**
- **SIC with full bandwidth versus FDMA with local scheduling**
- **Great advantages for FDMA at low node densities**

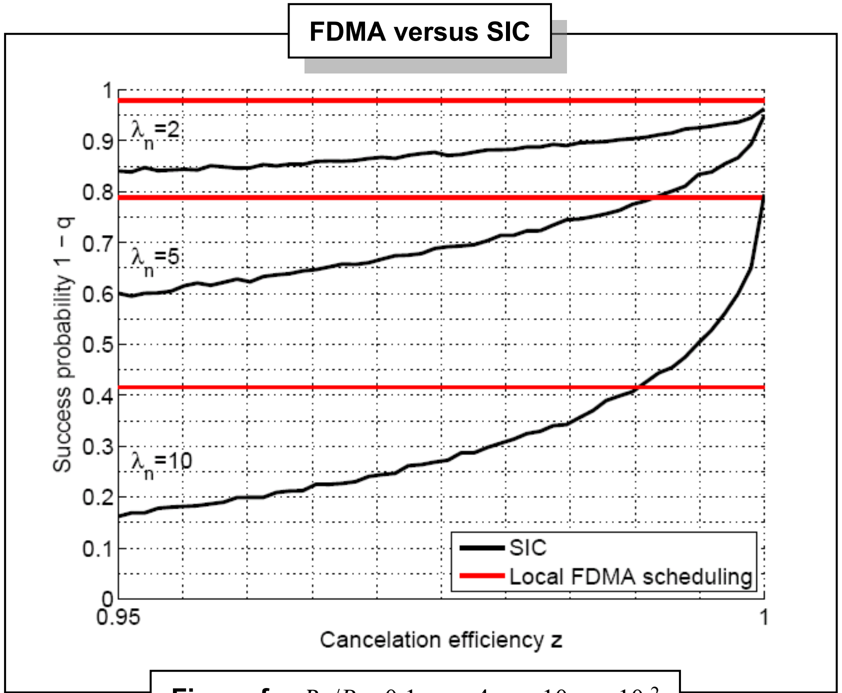
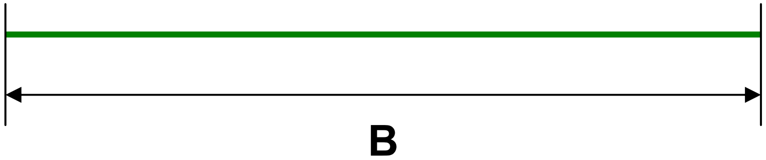
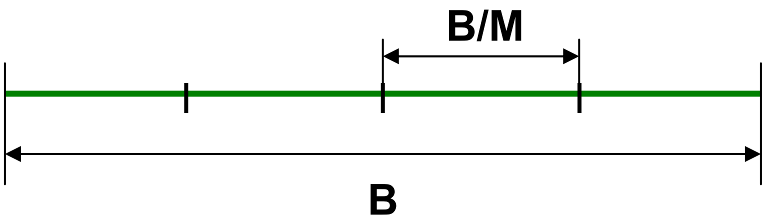


Figure for $R_m/B = 0.1$, $\alpha = 4$, $r = 10$, $\varepsilon_o=10^{-2}$





SIC model cf. Weber et al., *Transmission Capacity of Wireless Ad Hoc Networks with Successive Interference Cancelation*, IEEE Transactions on Information Theory, August 2007

On-going research

Algorithms for channel assignment in ad hoc networks and practical aspects.

Theoretical limits

1

- Further research on FDMA: Scheduling beyond the communication range
- Comparison with SIC, Rx/Tx-Scheduling, *Fading,* MIMO
- Including external interference in the model

FDMA in ad hoc networks

Practical algorithms

2

- Protocol design, PHY/MAC
- Graph coloring algorithms for channel selection
- Describing performance in the PPP model

Discussion / Q&A

Further questions, feedback appreciated:
jens.elsner@kit.edu