Upper and Lower Bound on Signal-to-Noise Ratio Gains for Cooperative Relay Networks

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***Abstract*— Cooperative networking as a means of creating spatial diversity is used in order to mitigate the adverse effect of fading in a wireless channel and increase reliability of communications. We investigate signal-to-noise ratio (SNR) gain in wireless cooperative networks. We show that the differential SNR gain in the high data rate regime, which we refer to as SNR gain exponent ζ_∞, is independent of the relaying strategy and only depends on the number of transmission phases used for communication. Furthermore, a straight-line upper and lower bound is derived based on geometric considerations. It is shown that the approximation error of the upper bound with respect to the exact SNR gain tends to zero for $R \to \infty$. For the lower bound, the approximation error tends asymptotically to a constant factor δ for $R \to \infty$. Both bounds are the best possible straight-line bounds with respect to absolute error.**

***Keywords*— cooperative communications, relaying, signal-to-noise ratio gain, upper bound, lower bound.**

I. Introduction

The randomness of the wireless channel leads to time-variant fluctuations of the received signal amplitude. This adverse effect of fading can be mitigated by diversity techniques [1], [2]. The creation of (spatial) diversity is one of the benefits of relaying and cooperation. In cooperative networks mobile nodes pool their resources in order to achieve a better performance, i.e., increase the reliability of communications. Along with those benefits come challenges for the design of large scale distributed networks such as cooperative relay networks. Those challenges can be settled in the field of network information theory (e.g., network coding) and communications theory (e.g., medium access, combining receiver structures).

There are several performance metrics for the evaluation of cooperative networks. Among the most important ones are achievable rate, diversity-multiplexing tradeoff (DMT) [3], coding gain [4], and signal-to-noise ratio (SNR) gain [5]. In this paper, we focus on the SNR gain of cooperative networks. We first focus on the straight-line upper bound and will then use found results in order to derive the straight-line lower bound. Indeed, it will be shown that the lower bound is not very convenient, but it is nonetheless the best linear lower bound possible.

The paper is structured as follows. In Section II we briefly summarize our main contributions. Section III introduces our system model. We will see that an "abstract" model is sufficient for the basis of our investigations. A detailed analysis on SNR gain is given in Section IV, whereas Section V deals with the derivation of the upper and lower bound. We give some numerical examples of very simple and well-understood cooperative schemes to show the tightness of our upper bound in Section VI before we conclude the paper with Section VII.

II. Main Contributions

We establish a unified framework in order to evaluate cooperative networks with respect to SNR gain. Noteworthy, that statements in this paper are valied for $R \geq 1$ bit/s/Hz which seems reasonable from a practical perspective. We introduce a new performance criterion named SNR gain exponent ζ_∞ which describes the slope of SNR gain for large values of R and thus the differential behavior of SNR gain. This new criterion is hence comparable to DMT introduced by Zheng and Tse in [3]. We show that in the high rate regime, i.e., for $R \to \infty$, the SNR gain exponent is only determined by the number of transmission phases L and independent of the required outage probability or channel conditions. SNR gain exponent can then be approximated by $\zeta_\infty \approx 3\,(1-L)$. Furthermore, we derive a straight-line upper and a straight-line lower bound on SNR gain. It is shown that these two bounds are the best possible straight-line bounds with respect to absolute error.

III. System Model

The channel gain between node i and j is denoted by h_{ij}. A vector $\mathbf{h}$ contains all channel gains between nodes in the network. For instance, consider a network with one source, one relay, and one destination. Then, $\mathbf{h}$ becomes $(h_{\mathrm{sd}}, h_{\mathrm{sr}}, h_{\mathrm{rd}})$. We consider an abstract channel model where we do not have to define a special probability distribution for the channel gains. Of course, when it comes to a more precise investigation, then a proper probability distribution must be chosen in order to describe channel properties. For instance, when we consider a quasi-static transmission scenario where the delay requirements are short compared to the coherence time of the channel, h_{ij} is modeled as a complex-valued circular-symmetric Gaussian random variable. Therefore, the magnitude $|h_{ij}|$ follows a Rayleigh distribution. The channel input of each node is limited to average power P. We assume white Gaussian noise to be added on each transmission path which follows $\mathcal{CN}(0, N)$. SNR is then given by $\mathsf{SNR} = P/N$.

We now define some basic notations that are used throughout the paper. R is the target rate in bit/s/Hz (which equals bit/channel use). SNR gain parametrized on outage probability is given by $\Delta(p_{\text{out}})$. If we express SNR gain in dB, we denote it by $\Delta_{\mathsf{SNR}}(p_{\text{out}})$. The variable L describes the number of transmission phases used for transmission. First order derivation of a function $f(x)$ is denoted by $f'(x)$ and second order derivation by $f''(x)$. All sums $\sum_i$ in this paper stand for $\sum_{i=0}^{L-1}$ if not stated otherwise.

IV. Signal-to-Noise Ratio Gain

A. Definition

In order to identify the benefits of various cooperation strategies upon direct transmission, we introduce the SNR gain. SNR gain of a cooperation scheme over direct transmission that achieves the same outage probability p_{out} is defined as

$$\Delta(p_{\text{out}}) \triangleq \frac{\mathsf{SNR}^{(\text{DT})}}{\mathsf{SNR}^{(\cdot)}}, \tag{1}$$

where the superscript (DT) describes the SNR of direct transmission and the superscript $(\cdot)$ stands for various relay strategies. SNR gain is a function of data rate R, channel gains $\mathbf{h}$, and outage probability p_{out}. With respect to our definition in (1), SNR gain can generally be factorized as

$$\Delta(R, \mathbf{h}, p_{\text{out}}) = \Delta_{\text{th}}(R) \cdot f(\mathbf{h}) \cdot g(p_{\text{out}}), \tag{2}$$

where

$$\Delta_{\text{th}}(R) = \frac{\mathsf{SNR}_{\text{th}}^{(\text{DT})}}{\mathsf{SNR}_{\text{th}}^{(\cdot)}} = \frac{2^R - 1}{2^{LR} - 1} \tag{3}$$

is the ratio of the required threshold SNRs and thus rate-dependent, $f(\mathbf{h})$ contains propagation conditions, and $g(p_{\text{out}})$ as a function of outage probability p_{out} is linked to diversity order. Expressed in dB we get

$$\Delta_{\mathsf{SNR}}(p_{\text{out}}) = 10 \cdot \log_{10}\left(\Delta(R, \mathbf{h}, p_{\text{out}})\right). \tag{4}$$

In the following, for the sake of description, we will simply use the notations Δ and Δ_{SNR}.

B. Geometric Construction

The derivation of our upper and lower bound is based on a simple geometric construction. Let SNR gain be a monotonically decreasing and concave function (both will be proved later) in R. A (straight-line) upper bound can then be derived by constructing a straight-line with the same slope as the SNR curve for $R \to \infty$ (it can easily be seen that the slope for $R \to \infty$ is the maximal slope of the SNR gain under these assumptions). Now, let this straight-line come from infinity. We approach the curve of SNR gain until our straight-line "lies upon" the curve of SNR gain. In order to derive the lower bound, we let the straight-line come from minus infinity until we "touch" the SNR curve in the point $Q = (R = 1~\text{bit/s/Hz}, \Delta_{\mathsf{SNR}})$. The principle of the geometric construction is depicted in Fig. 1. We see that once we know the upper bound and the value for $R = 1$ bit/s/Hz, construction of the lower bound is straightforward.

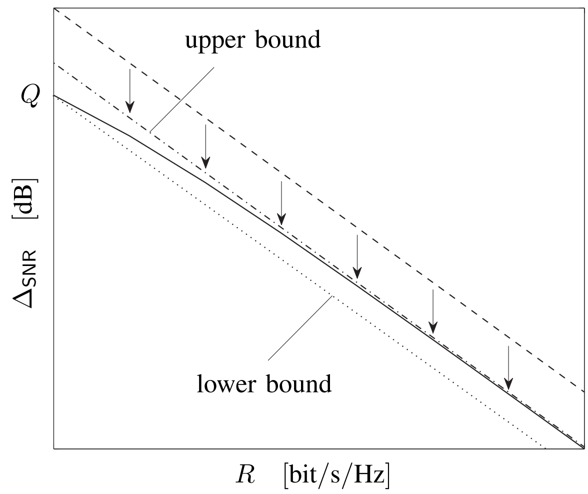

Fig. 1. Geometric construction of upper and lower bound.

C. Slope

As already stated, the geometric construction of the upper and lower bound require that SNR gain is a monotonically decreasing and concave function in R. We state the following lemma.

Lemma 1: SNR gain $\Delta_{\mathsf{SNR}}(R)$ is a monotonically decreasing function in R, that is:

$$\Delta'_{\mathsf{SNR}}(R) = \frac{\mathrm{d}}{\mathrm{d}R}\left(-\log_{10}\left(\sum_i 2^{iR}\right)\right) \leq 0 \tag{5}$$

Proof: Since $f(\mathbf{h})$ and $g(p_{\text{out}})$ are independent of rate R and applying the product rule for logarithms to (2), we get

$$\Delta'_{\mathsf{SNR}}(R) = \frac{\mathrm{d}}{\mathrm{d}R}\left(\log_{10}\left(\frac{2^R - 1}{2^{LR} - 1}\right)\right). \tag{6}$$

The factor 10 can be ignored as we are only interested in the sign of the derivative. The derivative can be directly calculated from (6), however, as we will see later, it is more convenient to express the argument by means of a geometric series. We have

$$\Delta'_{\mathsf{SNR}}(R) = \frac{\mathrm{d}}{\mathrm{d}R}\left(\log_{10}\left(\sum_i 2^{iR}\right)^{-1}\right) \tag{7}$$

$$= -\frac{\log_{10} e}{\sum_i 2^{iR}} \cdot \frac{\mathrm{d}}{\mathrm{d}R}\left(\sum_i 2^{iR}\right) \tag{8}$$

$$= -\log_{10} 2 \cdot \frac{\sum_i i\, 2^{iR}}{\sum_i 2^{iR}} \tag{9}$$

Since all summands of (9) are positive, $\Delta'_{\mathsf{SNR}}(R) \leq 0$ and Lemma 1 is proved. ■

A special case occurs when SNR gain is independent of R. This is the case for incremental relaying as introduced in [6] and transmit diversity [7].

The construction of the upper and lower bound requires knowledge of the slope for $R \to \infty$. This is also of particular interest as it gives information about the differential behavior of various cooperation protocols. For instance, for a specific target rate R, a cooperation strategy that consists of two relays might be preferable to a one-relay protocol. However, this may not be true for all values of R dependent on the chosen cooperation strategy. Therefore, it is advantageous to know the asymptotic behavior of SNR gain for various cooperation strategies in order to find the most suitable one.

From Fig. 1 we can conclude that

$$\Delta_{\mathsf{SNR}} \propto R^{\zeta_\infty},\ \zeta_\infty \leq 0, \tag{10}$$

where we define the SNR gain exponent as

$$\zeta_\infty \triangleq 3 \cdot \lim_{R\to\infty} \frac{\log_2 \Delta(R, \mathbf{h}, p_{\text{out}})}{R} \quad [\text{dB/bit/s/Hz}]\,. \tag{11}$$

The factor 3 is due to the manipulation of $\log_{10}$ into $\log_2$ which is preferable since we deal with expressions on mutual information.

Theorem 1: SNR gain exponent ζ_∞ is only a function of the number of transmission phases L and can be approximated by

$$\zeta_\infty \approx 3\,(1-L). \tag{12}$$

Proof: A comparison of (11) and (12) indicates that we have to show

$$\lim_{R\to\infty} \frac{\log_2 \Delta(R, \mathbf{h}, p_{\text{out}})}{R} = 1 - L. \tag{13}$$

We start with the general representation of $\Delta(R, \mathbf{h}, p_{\text{out}})$ in (2). Inserted in (11), we get

$$\lim_{R\to\infty} \frac{\log_2\left(\Delta_{\text{th}}(R)\cdot f(\mathbf{h})\cdot g(p_{\text{out}})\right)}{R} = \lim_{R\to\infty} \frac{\log_2\left(\Delta_{\text{th}}(R)\right)}{R}$$

since $f(\mathbf{h})$ and $g(p_{\text{out}})$ are not rate-dependent and corresponding terms vanish for $R \to \infty$. We see that the SNR gain exponent only depends on the ratio of the required threshold SNR values. This yields

$$\begin{aligned} \lim_{R\to\infty} \frac{\log_2\left(\Delta_{\text{th}}(R)\right)}{R} &= \lim_{R\to\infty} \frac{\log_2\left(\left(2^R-1\right)/\left(2^{LR}-1\right)\right)}{R} \\ &= 1-L \end{aligned}$$

which concludes the proof. ■

SNR gain exponent ζ_∞ [dB] depends linearly on the number of transmission phases L. Thus, we can conclude that the advantage in SNR gain of multi-phase transmission schemes is dominant for low-rate systems. This shows the tradeoff between the ability to transmit with a higher data rate and the reliability of communications.

D. Concavity

Lemma 2: SNR gain $\Delta_{\mathsf{SNR}}(R)$ is a concave function in R, that is:

$$\Delta''_{\mathsf{SNR}}(R) = \frac{\mathrm{d}^2}{\mathrm{d}R^2}\left(-\log_{10}\left(\sum_i 2^{iR}\right)\right) \leq 0 \tag{14}$$

Proof: In (9) the first derivative of Δ_{SNR} is given. Derivating again by applying quotient rule and neglecting positive constants which do not influence the sign yields

$$-\frac{\left(\sum_i i\,2^{iR}\right)' \sum_i 2^{iR} - \sum_i i\,2^{iR}\,\left(\sum_i 2^{iR}\right)'}{\left(\sum_i 2^{iR}\right)^2}. \tag{15}$$

SNR gain Δ_{SNR} is concave if $\Delta''_{\mathsf{SNR}} \leq 0$, which means that the nominator of the above equation satisfies

$$\left(\sum_i i\,2^{iR}\right)' \sum_i 2^{iR} \geq \sum_i i\,2^{iR}\,\left(\sum_i 2^{iR}\right)'. \tag{16}$$

After some manipulation we get

$$\sum_i i^2\,2^{iR} \sum_i 2^{iR} \geq \left(\sum_i i\,2^{iR}\right)^2. \tag{17}$$

This is exactly the Cauchy-Schwarz inequality $(\mathbf{a}^T\mathbf{a})(\mathbf{b}^T\mathbf{b}) \geq (\mathbf{a}^T\mathbf{b})^2$ with $a_i = i\sqrt{2^{iR}}$, $b_i = \sqrt{2^{iR}}$, and thus $a_i\,b_i = i\,2^{iR}$. This proves Lemma 2. ■

Remark 1: An equivalent way to prove that SNR gain is a concave function and that avoids the second derivative is to show that the first derivative is a decreasing function. Let ρ be an arbitrary number greater than or equal to 0 ($\rho \geq 0$). Then the following inequality must be satisfied:

$$\left(\frac{\mathrm{d}}{\mathrm{d}R}\Delta_{\mathsf{SNR}}\right)(R+\rho) \overset{!}{\leq} \frac{\mathrm{d}}{\mathrm{d}R}\Delta_{\mathsf{SNR}}(R) \tag{18}$$

With (9) this becomes

$$\frac{\sum_i i\,2^{i(R+\rho)}}{\sum_i 2^{i(R+\rho)}} \overset{!}{\geq} \frac{\sum_i i\,2^{iR}}{\sum_i 2^{iR}}. \tag{19}$$

The nominator on both sides increases faster than the corresponding denominator due to the factor i. Hence, condition (18) is met and SNR gain is a concave function.

Equality is achieved for $\rho = 0$ and/or $L = 1$. The latter corresponds to direct transmission. This is, however, immediately clear since SNR gain is defined according to the SNR of direct transmission. If ρ is fixed, equality is also achieved for $R \to \infty$.

V. Bounds

A. Upper Bound

Theorem 2: Let ζ_∞ [dB] be the SNR gain exponent for large values of rate R and be given by (12). Then SNR gain can be upper bounded with a straight line by

$$\Delta_{\mathsf{SNR}} \leq \zeta_\infty R + t^+ = \Delta^+_{\mathsf{SNR}}, \tag{20}$$

where t^+ is a function of channel realizations and outage probability and is given by

$$t^+ = 10\,\log_{10}\left(f(\mathbf{h})\cdot g(p_{\text{out}})\right). \tag{21}$$

Proof: Considering the generalized expression of SNR gain ((2) and (4)), we get

$$\Delta_{\mathsf{SNR}} = 10\,\log_{10}\Delta_{\text{th}}(R) + 10\,\log_{10}\left(f(\mathbf{h})\cdot g(p_{\text{out}})\right). \tag{22}$$

It can easily be seen that the second summand on the right-hand side is t^+. The first summand can be manipulated into:

$$\begin{aligned} 10\,\log_{10}\Delta_{\text{th}}(R) &= 10\,\log_{10}\frac{2^R-1}{2^{LR}-1} \\ &= -10\,\log_{10}\sum_i 2^{iR} \\ &= -10\,\log_{10}\left(1+2^R+\cdots+2^{(L-1)R}\right) \\ &\leq -10\,\log_{10} 2^{(L-1)R} \approx 3\,(1-L)\,R \end{aligned}$$

The second equality follows from a series representation equal to (7). Comparing the last equation to (12), Theorem 2 is proved. ■

Remark 2: It can easily be seen that this upper bound is the best straight-line upper bound with respect to absolute error that can be achieved. Consider the third equation above. In order to find our upper bound, we took the largest value of the sum in the argument of the logarithm and ignored the rest. Taking another value instead and ignoring all remaining summands would definitely lead to a straight-line upper bound that is less tight. Another possibility is to take the largest value and add 1 to it. This will definitely lead to a tighter bound. However, this is not a straight-line bound anymore. Hence, we see that our upper bound is really the tightest straight-line upper bound that can be found to SNR gain.

Our construction clearly points out that the upper bound converges asymptotically to the exact value of SNR gain. Accordingly,

$$\lim_{R\to\infty}\left(\Delta^{+}_{\mathsf{SNR}}-\Delta_{\mathsf{SNR}}\right)\to 0 \quad\Leftrightarrow\quad \lim_{R\to\infty}\frac{\Delta^{+}}{\Delta}\to 1. \tag{23}$$

The point $\Delta^{+}_{\mathsf{SNR}}(R=1)$ determines the most convenient straight line upper bound to SNR gain. It can easily be shown that

$$\Delta^{+}_{\mathsf{SNR}}(R=1)=10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2^{L-1}}\right). \tag{24}$$

B. Lower Bound

For the construction of the lower bound, we have to use the same slope as for the upper bound. Since SNR gain is generally a concave function, the lower bound cannot converge to the exact value of SNR gain. There will be a difference δ for high values of rate R. This yields

$$\lim_{R\to\infty}\left(\Delta_{\mathsf{SNR}}-\Delta^{-}_{\mathsf{SNR}}\right)\to \delta \quad\Leftrightarrow\quad \lim_{R\to\infty}\frac{\Delta}{\Delta^{-}}\to \tilde{\delta}=\text{const.} \tag{25}$$

with $\tilde{\delta}=10^{0.1\,\delta}$.

Theorem 3: Let $\zeta_\infty\,$[dB] be the SNR gain exponent for large values of rate R and be given by (12). Then SNR gain can be lower bounded with a straight line by

$$\begin{aligned}\Delta_{\mathsf{SNR}} &\geq \zeta_\infty R+t^{-}\\ &= \zeta_\infty(R-1)+\Delta_{\mathsf{SNR}}(R=1)=\Delta^{-}_{\mathsf{SNR}},\end{aligned} \tag{26}$$

where t^{-} is a function of channel realizations, outage probability and number of transmission phases and is given by

$$t^{-}=10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2\,(1-2^{-L})}\right). \tag{27}$$

Equality is achieved for $R=1$ bit/s/Hz.

Proof: The statement on equality follows directly from (26). As already mentioned, the lower bound is constructed by creating a straight line that includes $\Delta_{\mathsf{SNR}}(R=1)$ and possesses the slope of SNR gain for $R\to\infty$, namely ζ_∞. Then, from (26), we can see that $t^{-}=\Delta_{\mathsf{SNR}}(R=1)-\zeta_\infty$. We next calculate the value of SNR gain for $R=1$ bit/s/Hz and get

$$\begin{aligned}\Delta_{\mathsf{SNR}}(R=1) &= 10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{\sum_i 2^i}\right)\\ &= 10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2^L-1}\right),\end{aligned} \tag{28}$$

where the second equation follows from the presentation of the sum as a geometric series. Consequently,

$$\begin{aligned}t^{-} &= 10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2^L-1}\right)-3(1-L)\\ &= 10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2^L-1}\cdot\frac{1}{2^{1-L}}\right)\\ &= 10\,\log_{10}\left(\frac{f(\mathbf{h})\cdot g(p_{\mathrm{out}})}{2(1-2^{-L})}\right),\end{aligned} \tag{29}$$

which proves (27). ■

Corollary 1: The value t^{-} can be expressed in terms of t^{+} and L as

$$t^{-}=t^{+}-10\,\log_{10}(2(1-2^{-L})). \tag{30}$$

Proof: The proof follows directly by comparing (27) to (21). ■

We will show later that the difference between t^{+} and t^{-} is the maximal error of both bounds.

C. Approximation Error

In this subsection we investigate the tightness of our bounds. For that purpose, we apply the absolute error which can be expressed as

$$\epsilon^{\pm}_{\mathrm{abs}}=|\Delta^{\pm}_{\mathsf{SNR}}-\Delta_{\mathsf{SNR}}|. \tag{31}$$

As stated before, the limiting behavior of the upper bound is given by

$$\lim_{R\to\infty}\epsilon^{+}_{\mathrm{abs}}=0 \tag{32}$$

and for the lower bound we have

$$\lim_{R\to\infty}\epsilon^{-}_{\mathrm{abs}}=\delta. \tag{33}$$

Due to the construction of the upper and lower bound, it becomes immediately clear that the difference between these bounds denotes the maximal error. Accordingly,

$$\max\epsilon^{\pm}_{\mathrm{abs}}=\delta=\Delta^{+}_{\mathsf{SNR}}-\Delta^{-}_{\mathsf{SNR}}. \tag{34}$$

Inserting (20) and (26) in (34) and applying Corollary 1, we get

$$\delta=t^{+}-t^{-}=10\,\log_{10}(2(1-2^{-L})). \tag{35}$$

For two transmission phases this becomes $\delta\approx 1,76$ dB. Hence, $\tilde{\delta}=2(1-2^{-L})=1.5$. We will come back to this in the next section, where we give some simple examples in order to show the tightness of our bounds.

Fig. 2 illustrates the absolute error of the upper and lower bound over rate R. Due to the geometrical construction, both curves intersect for the value $\epsilon^{\pm}_{\mathrm{abs}}=\delta/2$.

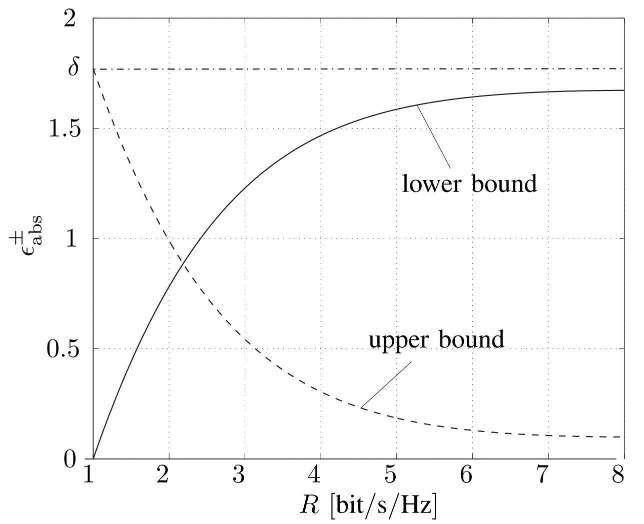

Fig. 2. Absolute error of upper and lower bounds with respect to real value.

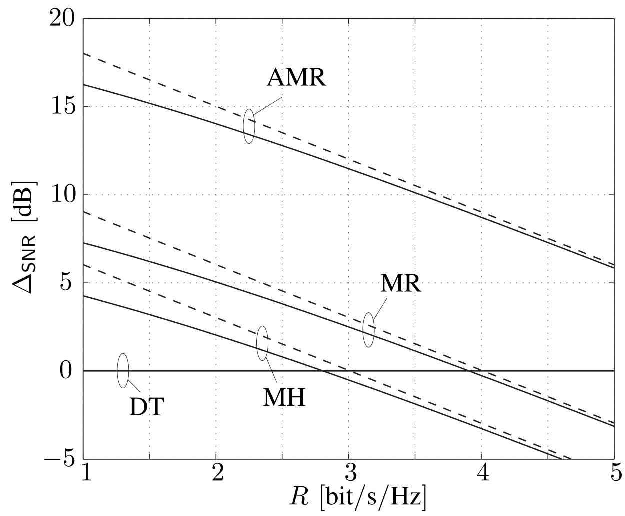

Fig. 3. SNR gains Δ_{SNR} for multi-hop (MH), multi-route (MR), and adaptive multi-route (AMR) relaying with upper bounds for $p_{\mathrm{out}} = 10^{-3}$ and $\alpha = 4$. Relay has been placed half-way between source and destination, i.e., $d_{\mathrm{sr}} = 0.5$. Direct transmission (DT) serves as reference base.

VI. Numerical Examples

We now deal with very simple examples to show the tightness of our bounds. We consider networks that consist of one source, one relay, and one destination. The relaying strategies are multi-hop, multi-route, and adaptive multi-route, where the relay only forwards source information if it has been able to decode properly. We have two transmission phases and thus $L = 2$. The path loss model is similar to the one used in [8], i.e., $\sigma^2_{ij} = d_{ij}^{-\alpha}$, where σ^2_{ij} is the mean value of $|h_{ij}|^2$, α is the path loss factor, and d_{ij} denotes the distance between node i and j. The source-destination distance has been normalized to 1 and $d_{\mathrm{rd}} = 1 - d_{\mathrm{sr}}$. For a deeper anaylsis on the derivation of the following bounds, we refer the reader to [9].

For multi-hop relaying, we derive the following upper bound:

$$\Delta^{(\mathrm{MH})}_{\mathsf{SNR}} \leq -3R - 10\log_{10}(d^{\alpha}_{\mathrm{sr}} + (1 - d_{\mathrm{sr}})^{\alpha}) \qquad (36)$$

The upper bound for multi-route relaying eventually becomes after some algebraic manipulation

$$\Delta^{(\mathrm{MR})}_{\mathsf{SNR}} \leq -3R - 10\alpha \log_{10} d_{\mathrm{sr}}. \qquad (37)$$

Note that in both cases the upper bound, and thus SNR gain in general, are independent of outage probability. This is, however, not the case for adaptive multi-route relaying. Here, the upper bound becomes

$$\Delta^{(\mathrm{AMR})}_{\mathsf{SNR}} \leq -3R - 5\log_{10} \frac{(d^{\alpha}_{\mathrm{sr}} + (1 - d_{\mathrm{sr}})^{\alpha})p_{\mathrm{out}}}{2}, \qquad (38)$$

which shows a $\log_{10} p_{\mathrm{out}}$ behavior.

All three cases with their exact SNR gains and the upper bounds are illustrated in Fig. 3. The lower bounds have been omitted for the sake of presentation. For simulations, outage probability was $p_{\mathrm{out}} = 10^{-3}$, the relay has been placed half-way between source and destination and the path loss factor was set to $\alpha = 4$.

VII. Concluding Remarks

In this paper we investigated the SNR gains of cooperative communications over direct transmission. We showed that the differential SNR gain, which we introduced as SNR gain exponent ζ_{∞}, is independent of the relaying strategy in the high data rate regime and only depends on the number of transmission phases L. It can be approximated by $\zeta_{\infty} \approx 3(1 - L)$. Therefore, multi-phase transmission schemes are advantageous for low-rate systems. We further derived a straight-line upper and lower bound which are both the best straight-line bounds that can be achieved with respect to the absolute error to the exact curve.

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