Capacity Regions of Wireless Multi-Channel Ad Hoc Networks

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***Abstract*—We consider the influence of FDMA on the capacity region of wireless ad hoc networks with a small number of nodes. It is found that FDMA offers an additional degree of freedom that is beneficial in interference-limited networks. Furthermore, simulation results indicate that static FDMA channel assignment over a time division schedule can achieve rates comparable to dynamic channel assignment.**

I. Introduction

The capacity of wireless networks with arbitrary geometry is not yet well understood. To study the design trade-offs in networks with a small number of nodes, Toumpis and Goldsmith introduced the concept of capacity regions [1][1]. This approach yields some insight on the gains of multi hop routing, power control and successive interference cancellation in ad hoc networks. An aspect not yet studied in this framework is the question of the influence of FDMA. This is especially of interest as today's wireless receivers are highly flexible and offer a large tuning bandwidth but are limited in their system bandwidth due to high frequency design considerations such as desired dynamic range, power consumption and co-site interference.

In this paper, we extend the model of Toumpis and Goldsmith and examine the influence of FDMA on the capacity region. We further show the performance gains achievable by optimally assigning channels in a network.

The remainder of this paper is structured as follows. Section II introduces the system model. Parameter dependencies of transmission rates in the system model are discussed in Section III, while Section IV discusses the influence of FDMA on the capacity region. Section V concludes.

II. System Model

The network under study consists of n nodes $A_1, A_2, ..., A_n$. The network bandwidth W is the total bandwidth available for communication and is split into M orthogonal channels of system bandwidth $W_m = \frac{W}{M}$. Communication between the nodes is always directed, broadcasting is not allowed. Nodes operate in half-duplex mode, i.e., they cannot transmit and receive simultaneously. All nodes have complete knowledge of all network parameters.

Every node transmits at a fixed transmission power level P[2]. The received signal power is only determined by path loss and therefore proportional to $d_{ij}^{-\alpha}$ for $d_{ij} \geq 1$[3] where d_{ij} is the distance between node A_i and A_j and α is the path loss exponent. Thermal noise and background interference at the receivers are modeled as additive white Gaussian noise (AWGN) with power spectral density η. Furthermore, all unwanted interfering signals are treated as AWGN.

The transmission rate is based on the signal to interference and noise ratio (SINR) at the receiver A_j which is receiving information from A_i. Considering a a set of nodes which are transmitting at a specific time $\{A_t : t \in \tau\}$ the SINR at node A_j is given as

$$\gamma_{ij} = \frac{P d_{ij}^{-\alpha}}{\eta W_m + \sum_{k \in \tau, k \neq i} P d_{kj}^{-\alpha}}. \tag{1}$$

The transmitting node is assumed to adapt its transmission rate based on the SINR to meet a given performance metric, which is in our case based on the Shannon capacity. The transmission rate is hence

$$r_{ij} = W_m \log_2(1 + \gamma_{ij}) \,. \tag{2}$$

A. Transmission Schemes, Basic Rate Matrices and Time Division Schedule

A transmission scheme S describes the state of the network at a given time instance. It contains all active pairs of transmitters and receivers at a given time and the sources of information. The information flow in the network can be modelled by considering a set of transmission schemes. A time division schedule τ is the weighted combination of several transmission schemes and given by $\tau = \sum_{i=1}^{k} a_i S_i$ with positive weights a_i.

A rate matrix is a mathematical representation of the data flow of a transmission scheme. It is a $n \times n$ square matrix whose elements are defined as

$$R_{ij} = \begin{cases} +r_{ij} & \text{if node } A_j \text{ is receiving at rate } R \\ -r_{ij} & \text{if node } A_j \text{ is transmitting at rate } R \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The row index i corresponds to the source of information, which, in case of multi hop routing, is not necessarily the transmitting node. Transmissions where the transmitting node is also the source of information have a negative entry on the diagonal. The matrix can be written as a $n \cdot (n-1)$ dimensional vector $\rho = R_{uv}, i = 1, ..., n \cdot (n-1)$ so that every element

[1]This paper is a straight forward extension, please see [1] for related work, references and model justifications.

[2]As shown in [1] the gain in capacity by varying the transmission power for variable transmission rates is negligible.

[3]All distances are without loss of generality assumed to be greater than 1 unit to assure physical plausibility of the model.

of the vector gives the rate of one possible transmission. The row and column indices are given by

$$u = \begin{cases} \lceil \frac{i}{n} \rceil + 1 & \text{for } \lceil \frac{i}{n} \rceil < i - \lfloor \frac{i}{n} \rfloor n \\ \lceil \frac{i}{n} \rceil & \text{for } \lceil \frac{i}{n} \rceil > i - \lfloor \frac{i}{n} \rfloor n \end{cases} \tag{4}$$

and

$$v = \begin{cases} i - \lfloor \frac{i}{n} \rfloor n + 1 & \text{for } \lceil \frac{i}{n} \rceil < i - \lfloor \frac{i}{n} \rfloor n \\ i - \lfloor \frac{i}{n} \rfloor n + 2 & \text{for } \lceil \frac{i}{n} \rceil > i - \lfloor \frac{i}{n} \rfloor n \end{cases} . \tag{5}$$

The rate matrix of a time division schedule is equal to the weighted sum of the rate matrices of the associated transmission schemes: $R\left(\sum_{i=1}^{N} a_i S_i\right) = \sum_{i=1}^{N} a_i R(S_i)$. To be able to compare different transmission protocols a unit time interval is considered; $\sum_{i=1}^{N} a_i = 1$.

For a given transmission protocol there is a fixed set of possible transmission schemes. The set of rate matrices corresponding to these schemes is called the set of basic rate matrices, which span a $n(n-1)$ dimensional space of transmission rates. The convex hull of this set, without the physically unreasonable transmission schemes in which transmitting nodes transmit less information than the receiving nodes receive (indicated by negative off-diagonal elements in the rate matrix), is the capacity region C. In vectorial notation we write

$$C = \left\{ \sum_{i=1}^{N} a_i \rho(S_i) : a_i \geq 0, \sum_{i=1}^{N} a_i = 1 \right\} \cap V_n. \tag{6}$$

V_n denotes the set of all possible $n \cdot (n-1)$ dimensional vectors without negative elements.

B. Capacity Region and Uniform Capacity

A metric is needed to quantify the performance of a transmission protocol with a single number. In an ad hoc network, where all nodes can be sources and sinks of information, the uniform capacity is such a figure. The uniform capacity C_u of a given capacity region is the maximum aggregate communication rate that can be achieved by every transmission, so that every node communicates with every other node at a rate $r_{\max}$. When $r_k = \sum_{i=1}^{N} a_i R_k(S_i),\ \sum_{i=1}^{N} a_i = 1$ describes the rate of the k^{th} transmission of a point $r^d \in C$ with $R_k(S_i)$ denoting the element of the i^{th} rate vector corresponding to the k^{th} transmission, the point $r^{\text{opt}} \in C$ whose l_1-norm is the uniform capacity is

$$r^{\text{opt}} = \max\left\{ r \in C | r_k = r_l,\ \forall k, l = 1, ..., n \cdot (n-1) \right\}, \tag{7}$$

so that $r_{\max} = r_i^{\text{opt}},\ \forall i = 1, ..., n \cdot (n-1)$. The uniform capacity is then defined as

$$C_u = \sum_{k=1}^{n \cdot (n-1)} r_k^{\text{opt}}. \tag{8}$$

Geometrically this can be interpreted as the l_1-norm of the position vector of the intersection between the hull of the capacity region with a line of unit slope in all dimensions. The uniform capacity is determined by approaching the capacity region along the line of unit slope in all dimensions. To check whether a point r^d is inside or outside the capacity region a linear program

$$\min\{\sum_{i=1}^{N} a_i | \Psi a = r^d, 0 \leq a \leq 1\} \tag{9}$$

has to be solved, where Ψ denotes the horizontally concatenated basic rate matrices in vector notation. This way, the uniform capacity can be determined by iterative linear optimization.

C. Single Channel Rate Matrices

The shape of the capacity region depends on the transmission protocol that is used. In the following we will consider two different protocols - single hop routing and multi hop routing - with and without spatial reuse. In single hop routing without spatial reuse only one node is permitted to transmit at each time instance. If spatial reuse is allowed, parallel but interfering transmissions are permissible. In multi hop routing without spatial reuse information is allowed to be forwarded by one or more intermediate nodes but still only one transmission can take place at each time instance. The most complex protocol considered is multi hop routing with spatial reuse where information is forwarded and parallel transmissions are permitted.

With increasing complexity of the protocols the capacity region grows as shown in Fig. 2.

Note that, in the single channel case, each transmission scheme has exactly one associated basic rate matrix.

D. Multi Channel Rate Matrices

If multiple channels are used, a given transmission scheme is associated with more than one rate matrix. Each of these rate matrices is created by a different receive channel assignment. For n nodes and M channels there are M^n possibilities to assign channels to nodes from which many are interchangeable. Let $B_M^n = \{(x_1, ..., x_n) | x_i \in \{1, ..., M\}\},\ i = \{1, ..., n\}$ be the set of all possible variations, where x_i denotes the number of the receive channel of the i^{th} node with $|B_M^n| = M^n$. Since all channels are assumed to have the same properties such as bandwidth or noise power density, the first node can be assigned to a fixed channel and $|B_M^{n-1}| = M^{n-1}$. Each element $m_j = (1, x_2 ..., x_n),\ j = 1, ..., M^{n-1}$ of B_M^{n-1} is transformed to a matrix

$$K_j = \begin{pmatrix} 1 & K(1,2) & \cdots & K(1,n) \\ 0 & K(2,2) & \cdots & K(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & K(M,2) & \cdots & K(M,n) \end{pmatrix} \tag{10}$$

with every $K(l, o) = 1$ if the o^{th} channel is assigned to the l^{th} node. The sum of a column $\sum_{o=1}^{M} K(l, o),\ \forall o = 1, ..., n$ has to be 1, since one node can only receive in one channel. If μ_{jl} denotes the l^{th} row of K_j the set $B_U(n, M)$ now denotes the subset of B_M^{n-1} with

$$B_U(n,M) = \{K_j | K_j, K_o \in B_M^{n-1},\ \mu_{jl} \neq \mu_{op}, \\ \forall j, o = 1, ..., K^{n-1} \wedge j \neq o,\ l, p = 1, ..., M\}, \tag{11}$$

so that all variations are left out in which the same nodes have a common receive channel. Finally, the minimal set of distinguishable variations $B_S(n,M)$ is calculated by neglecting all variations of B_U in which a channel is not used at all:

$$B_S(n,M) = \{K_j \in B_U | \sum_{o=1}^{n} K_j(l,o) > 0,\ l = 1, ..., M\}. \tag{12}$$

The capacity region and accordingly the uniform capacity depend on the chosen channel assignment.

III. Parameter Dependencies

Before analyzing simulation results in Section IV the principal parameter dependencies of the system model are shown by considering point-to-point transmission rates.

When splitting the network bandwidth into M channels the maximal rate a transmission can achieve decreases according to (2). On the other hand, if two transmissions that are active in a transmission scheme use different channels, their rates increase as they do not interfere with each other. Fig. 2 shows a two-dimensional slice of the capacity region in the single and multi channel case for the topology of Fig. 1(a).

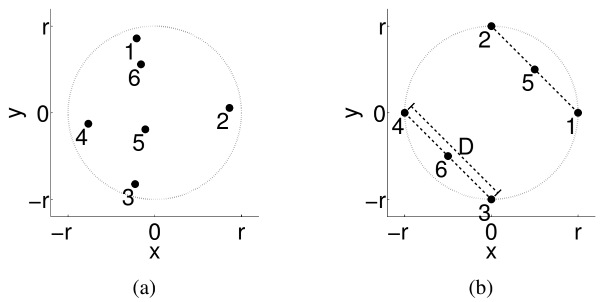


Fig. 1. a) Network with arbitrarily distributed nodes, $r = 1000$ m, $\alpha = 4$, $P = 0.1$ W, $W = 20$ MHz, $T = 293$ K, $\eta \approx 4 \cdot 10^{-21}$ W. b) Circular network topology for testing the parameter dependency of the transmission rates.

From curves a) and c) it can be seen that FDMA with single hop transmission yields a slight gain in transmission rates for certain operating points. In the multi hop case, curves d) and e), this gain can be even greater. On the other hand, the maximum rate achievable for the better link r_{12} decreases due to less available bandwidth.

The shape of the capacity region is highly dependent on the network topology.

A. Receiver noise versus interference

The capacity region has the form as shown in Fig. 2 only when noise power at the receiver is in the order of magnitude of the interference power due to spatial reuse. The interference power due to spatial reuse depends on the network diameter. In

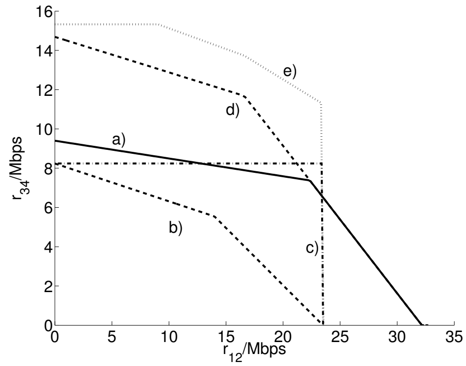


Fig. 2. Two-dimensional capacity region for different transmission protocols: a) Single hop routing, spatial reuse, one channel with bandwidth W. b) Single hop routing, spatial reuse, one channel with bandwidth $W/2$. c) Single hop routing, spatial reuse, two channels with bandwidth $W/2$. Both transmissions in different channels. d) Multi hop routing, spatial reuse, one channel with bandwidth W. e) Multi hop routing, spatial reuse, two channels with bandwidth $W/2$. Both transmissions in different channels.

the following, we consider the simple network topology shown in Fig. 1(b) with variable network radius r. We further assume thermal noise density $\eta = kT \approx 4 \cdot 10^{-21} \frac{\text{W}}{\text{Hz}}$ at $T = 293$ K[4], path loss exponent $\alpha = 4$, system bandwidth $W = 20$ MHz and transmission power level $P = 0.1$ W.

To show the influence of the ratio of interference and noise power at the receiver, we assume the most simple topology of nodes 1 to 4 as shown in Fig. 1(b). The achievable transmission rates for varying network radius r are depicted in Fig. 3(a).

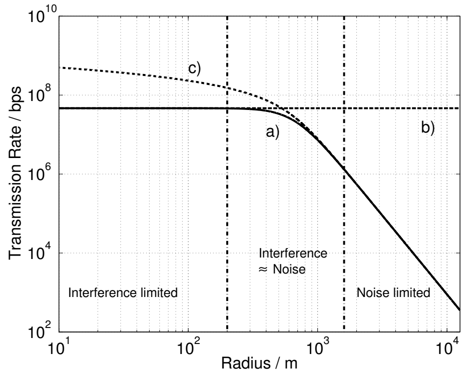
We define transmission rates to be interference limited if neglecting noise does not reduce the transmission rate by more than 1 percent, and vice versa to be noise limited, if neglecting interference does not reduce the transmission rate by the same amount.

B. Node positions and relaying

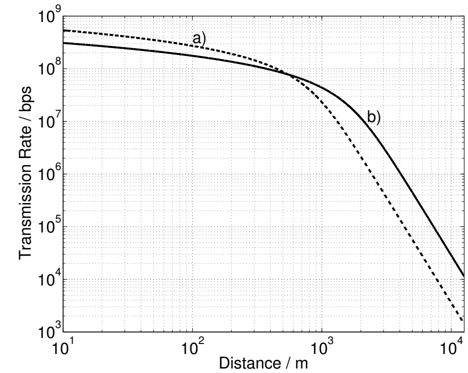
The gain in transmission rates that can be achieved in the multi hop regime also depends on the network size. For small distances between transmitter and receiver a direct transmission might even result in a higher data rate. If the distance between the nodes is large enough, the optimal relay position for transmissions with constant transmission power is centered on the line between the communicating nodes and a deviation from this position decreases the transmission rate.

If the relaying node deviates horizontally from its optimal position, the distance to either the transmitter or the receiver increases while the distance to the other decreases. With t_1 and t_2 denoting the time slices and r_1 and r_2 the transmission rates achieved by the transmission between transmitter and relay and between relay and receiver, respectively, the condition for causality $r_1 t_1 = r_2 t_2$ has to hold. We determine the sum

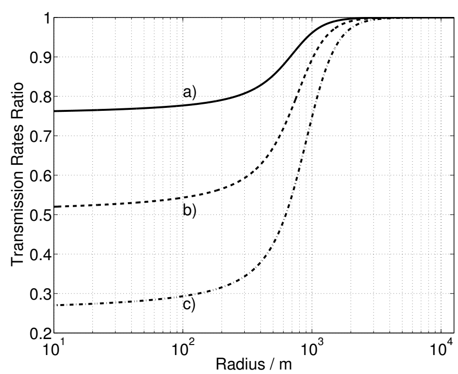
[4]In [1] the authors choose a very high noise power density of $10^{-10} \frac{\text{W}}{\text{Hz}}$ without any further justification. We choose to vary the radius r since it has a straight forward physical interpretation. A transmission rate between two nodes over a distance of 10 meters as in [1] is $r_1 = 0.14$ Mbps with $\eta = 10^{-10} \frac{\text{W}}{\text{Hz}}$. With $\eta = kT$ the same rate is achieved at a distance of about 2500 meters.



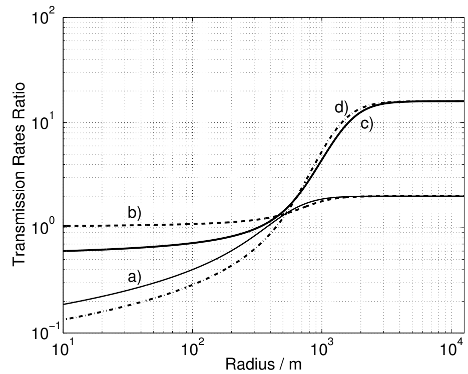
(a) Transmission rate depending on r. a) Exact. b) Noise neglected. c) Interference neglected.



(b) Transmission rate depending on the distance between transmitter and receiver. a) Direct transmission. b) Indirect transmission.



(c) Ratio of transmission rates with different system bandwidth, normalized to transmission with $W = 20$ MHz. a) $W = 15$ MHz. b) $W = 10$ MHz. c) $W = 5$ MHz.



(d) Ratio transmission rates with different transmission protocols, normalized to transmission rate with TDMA without spatial reuse and single hop routing. a) Spatial reuse, single hop routing. b) FDMA, single hop routing. c) FDMA, multi hop routing. d) Spatial reuse, multi hop routing.

Fig. 3. Parameter dependencies

$f(p) = t_1 + t_2$ of the time slices that are needed to transport a fixed amount of data R depending on the relay position p for fixed distance D between transmitter and receiver:

$$f(p) = \frac{R}{W \log_2\left(1 + \frac{Pp^{-\alpha}}{\eta W}\right)} + \frac{R}{W \log_2\left(1 + \frac{P(D-p)^{-\alpha}}{\eta W}\right)}. \tag{13}$$

This function has an optimum at $D/2$. For $D = 1000$ m, as shown in Fig. 4, this is a minimum. If the distance D between transmitter and receiver is below a minimal distance the central position leads to a maximum and direct transmission leads to a higher rate.

Fig. 3(b) compares the transmission rates of a direct transmission to an indirect transmission over a relay node placed at the optimal position depending on the distance between the nodes $D = \sqrt{2}r$ to determine the minimal distance between transmitter and receiver. As can be seen, multi hop routing only increases transmission rates if the distance between the communicating nodes is at least 600 meters.

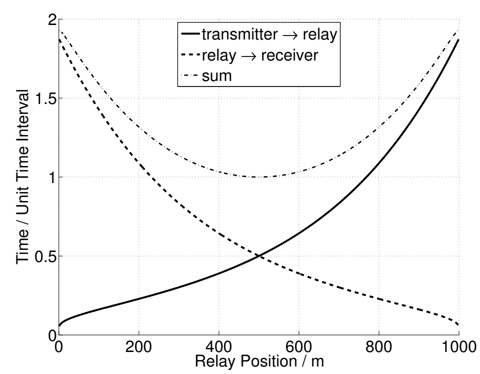


Fig. 4. Time slices needed to match rate with central relay position.

C. System Bandwidth

Furthermore, the radius r influences the impact of decreasing the system bandwidth on the transmission rate. Fig. 3(c) shows that for increasing r the transmission rate tends to be less dependent on the bandwidth, as the receiver moves to operate in a noise-limited regime.

D. Comparing Transmission Protocols

We compare achievable transmission rates of several transmission protocols in Fig. 3(d). The rates are normalized to the transmission rate achieved with TDMA and single hop routing. As can be seen, avoiding interference with TDMA or FDMA leads to better transmission rates for small r, i.e. when the rates are interference limited. For increasing r the influence of interference becomes smaller and spatial reuse achieves better rates since both transmissions can be active over the whole time interval (b). The gain of FDMA over TDMA also increases for higher r since influence of the bandwidth decreases (a).

In (c) and (d) data is relayed over the additional nodes 5 and 6 that lie in the middle of each transmitter-receiver pair. Here, the influence of the network radius on the multi hop rate gain as discussed before can be seen[5].

For small r transmission rates are interference limited and noise can be neglected. The ratio $r^{\text{SH}}_{\text{FDMA}}/r^{\text{SH}}_{\text{TDMA}}$ converges to 1 since the SINR at the receiver is then the same. With TDMA both transmissions are active half of the time with the full bandwidth, while FDMA transmissions are active the whole time interval with half the bandwidth. Accordingly, both rates are $r_{12} = r_{34} = W/2 \log_2(1+\gamma)$. For increasing r the limit is

$$\lim_{r\to\infty} \frac{r^{\text{SH}}_{\text{FDMA}}}{r^{\text{SH}}_{\text{TDMA}}} = \lim_{r\to\infty} \frac{W \log_2\left(1+\frac{P(d_{\text{SH}}(r)^{-\alpha})}{\eta \frac{W}{2}}\right)}{W \log_2\left(1+\frac{P(d_{\text{SH}}(r)^{-\alpha})}{\eta W}\right)} = 2, \quad (14)$$

since in the noise limited domain the bandwidth has no impact on the transmission rate. In that case the transmission rates of TDMA and FDMA are equal but with TDMA each transmission is active only half the time. The same explanation holds for the ratio of spatial reuse and TDMA.

The value of the limit of the ratio of multi hop FDMA and spatial reuse to TDMA is caused by the position of the relaying node and the path loss exponent. The limit is

$$\lim_{r\to\infty} \frac{r^{\text{MH}}_{\text{FDMA}}}{r^{\text{SH}}_{\text{TDMA}}} = \lim_{r\to\infty} \frac{\frac{W}{2} \log_2\left(1+\frac{P(d_{\text{MH}} r)^{-\alpha}}{\eta \frac{W}{2}}\right)}{W \log_2\left(1+\frac{P(k d_{\text{SH}}(r)^{-\alpha})}{\eta W}\right)} = \left(\frac{d_{\text{SH}}}{d_{\text{MH}}}\right)^{\alpha}. \quad (15)$$

Since the relaying node is centered between the communicating nodes this ratio is 2 and $r^{\text{MH}}_{\text{FDMA}}/r^{\text{SH}}_{\text{TDMA}} = 16$.

E. Fading

As shown and further studied in [1], time-varying flat-fading channels are modelled by dividing the unit time interval into F different fading states of the same length. With fading channels the channel gain between two nodes is $\chi_{ij} d_{ij}^{-\alpha}$ where χ_{ij} denotes the fading gain factor which is a Rayleigh-distributed random variable[6]. The channel gain matrix G containing all channel gains between the nodes varies for every fading state and leads to different transmission rates and therefore to a different set of basic rate matrices. The number of rate matrices increases by the factor F while each transmission scheme cannot be active for longer than the length of a fading state and the sum of the weights corresponding to one fading state is also limited to $1/F$.

[5] The distance between transmitter and receiver depends linear on the network radius.

[6] In [1] log-normal distributed shadowing factors are used.

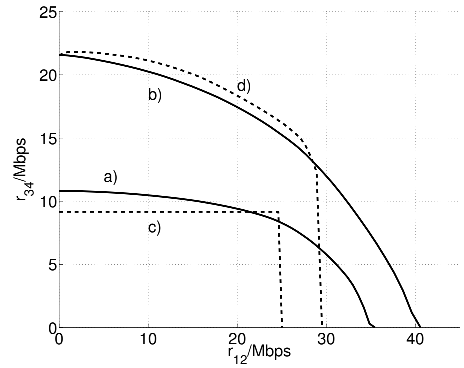


Fig. 5. Two-dimensional capacity region with 100 fading states. a) Single hop routing, spatial reuse. b) Multi hop routing, spatial reuse. c) Single hop routing, FDMA. d) Multi hop routing, fading.

Comparing Fig. 5 and Fig. 2, it can be seen that fading decreases the gain of the transmission rates achieved by FDMA.

IV. FDMA Capacity Regions

For channel assignment, two cases can be differentiated: dynamic and static channel assignment. For dynamic channel assignment, we allow nodes to change their receive channel over the course of a time division schedule. Alternatively, for static channel assignment we assume that each node employs the same receive channel over a time division schedule.

A. Computational issues

The complexity of computing the uniform capacity is highly dependent of the number of nodes in the network. The number of basic rate matrices N for multi hop routing and spatial reuse with n nodes is[7]

$$N = \sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{i} \binom{n-i}{i} i!(n-1)^i + 1. \quad (16)$$

Table I shows the number of basic rate matrices for up to 8 nodes. The complexity of computation to determine the

n	basic rate matrices single hop	basic rate matrices multi hop
3	7	13
4	25	145
5	81	1.041
6	331	19.651
7	1.303	196.813
8	5.937	5.227.713

TABLE I
NUMBER BASIC RATE MATRICES DEPENDING ON n.

[7] The formula given in [1] is incorrect.

uniform capacity depends linearly on the number of basic rate matrices and hence factorially on the number of nodes in the network.

Also, the number of possible channel assignments increases with the number of nodes as shown in Table II.

n	M	$\lvert B_S(n,M)\rvert$
3	2	3
3	3	1
4	2	7
4	3	6
4	4	1
5	2	15
5	3	25
5	4	10

TABLE II
NUMBER $|B_S(n,M)|$ OF UNIQUE CHANNEL ASSIGNMENTS DEPENDING ON n AND M.

These computational issues limit the evaluation of the system model to a small number of nodes.

B. Influence of the topology

We determined the uniform capacity of several different network topologies for up to 6 nodes. Nodes are placed in a circle with radius r. Nodes equally spaced on the edge of the circle form a circular network. In linear networks, the nodes are equally spaced on the diagonal.

Differences between the results depending on the topologies are due to the different distance distributions. For circular networks, the minimum and maximum distance can be determined as $d_{\text{min}} = \sqrt{2}r\sqrt{1-\cos\frac{2\pi}{n}}$, $d_{\text{min}} = \sqrt{2}r\sqrt{1-\cos\left(\lfloor\frac{n}{2}\rfloor\frac{2\pi}{n}\right)}$. For linear networks, $d_{\text{min}} = \frac{2r}{n-1}$, $d_{\text{max}} = 2r$.

For a small number of nodes, the difference between d_{min} and d_{max} in circular networks is small compared to the difference in linear networks. In linear networks links with transmission rates that are rather interference limited and links with transmission rates that are rather noise limited coexist.

For up to 6 nodes, linear and circular topologies can be regarded as extremes of other topologies.

C. Dynamic Channel Assignment

Fig. 6(a) and Fig. 6(b) show the results for circular networks with a radius of 100 meters and 1000 meters for single hop and multi hop routing. In both cases nodes are allowed to use spatial reuse and FDMA.

In the smaller network, multi hop routing does not lead to a gain in uniform capacity, the curves coincide with single hop routing. In this case, dividing the system bandwidth yields a slight gain but only until the number of channels reaches $\lfloor\frac{n}{2}\rfloor$. This is the maximum number of transmissions that can be active at the same time.

In the network with $r = 1000$m the gain due to FDMA is greater for single hop routing. For multi hop routing, FDMA leads to a smaller uniform capacity. With single hop routing, the communication of nodes that are farthest apart - and hence limits the uniform capacity - is strongly influenced by interference from concurrent transmissions. With multi hop routing this situation can be avoided and the influence of interference is mitigated. As seen in Fig. 3(d), in the multi hop case, spatial reuse achieves higher rates than FDMA. A division of the bandwidth reduces the uniform capacity.

With higher r the uniform capacity becomes increasingly independent of the bandwidth.

Fig. 6(c) and Fig. 6(d) show the results for linear networks with $r = 100$ m and $r = 1000$ m. For $r = 100$ m and an even number of nodes the uniform capacity for multi hop routing is higher than for single hop routing although the network radius is small. This is again due to the definition of the uniform capacity where the communication between the nodes with the largest distance is the limiting factor. With multi hop routing, the individual rates are smaller but the achieved uniform capacity is higher.

D. Static Channel Assignment

Fig. 6(e) and Fig. 6(f) show the uniform capacity for all unique static channel assignments compared to dynamic channel assignment for the topology shown in Fig. 1(b) with $r = 100$ m and $r = 1000$ m.

It can be seen that in the network with small distances between nodes the uniform capacity for different channel assignments has a wider spread than for the network with a radius of 1000 meters. Similar to the case of dynamic channel assignment, this is caused by the higher impact the division of the bandwidth has on the uniform capacity in interference limited networks.

The grouping of values reflects the number of possible distributions of nodes to channels.

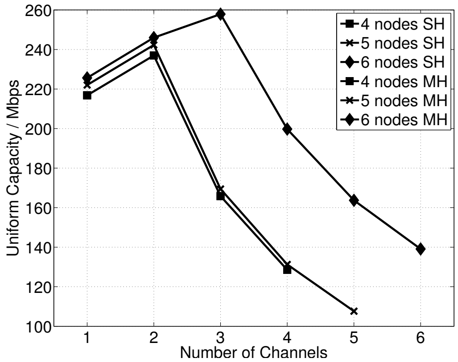
For networks with a small number of nodes the static assignment of receive channels is sufficient, since the uniform capacity that can be achieved is very close to the case of dynamic channel assignment. In this case, a network protocol can focus on scheduling the receive channels as there is no gain in scheduling transmissions globally.

The uniform capacity with dynamic channel assignment for the arbitrary network differs from the case of dynamic channel assignment for circular and linear topologies. This is due to the distribution of distances in the arbitrary network, which have an even greater spread.

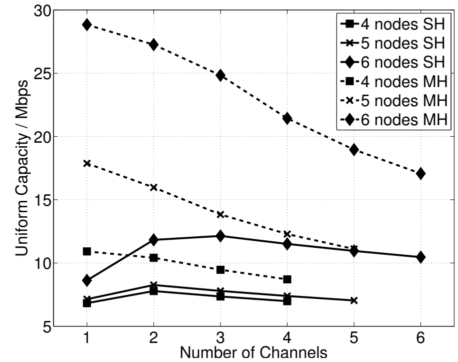
V. CONCLUSION

For the small networks considered here, we conclude that interference avoidance by splitting the operating bandwidth into orthogonal channels increases the capacity region and uniform capacity if interference is dominant in the network. In such networks, interference has a major impact and avoiding it via TDMA as well as FDMA is beneficial.

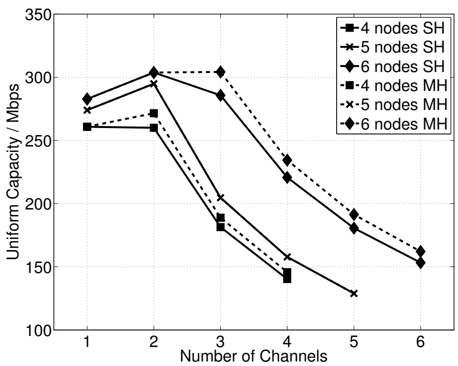
In case of single hop routing, the advantage is more pronounced and FDMA is advantageous even if noise and interference power are of the same order of magnitude. In noise limited networks spatial reuse achieves higher rates since the influence of internal interference can be neglected.



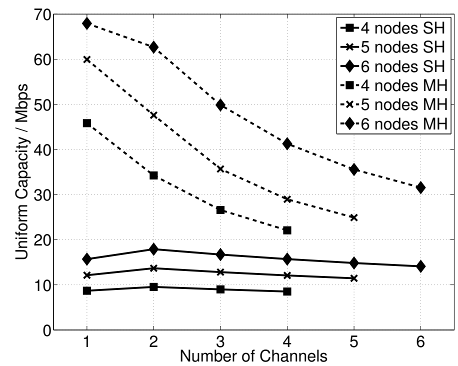
(a) Uniform capacity circular network for $r = 100$ m.



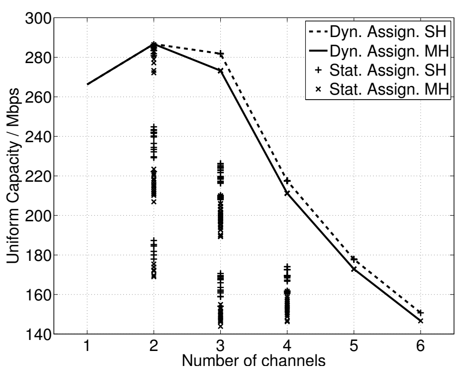
(b) Uniform capacity circular network for $r = 1000$ m.



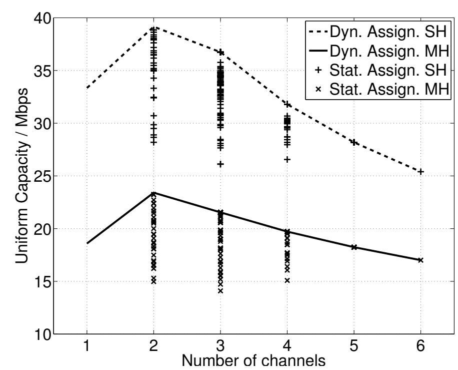
(c) Uniform capacity linear network for $r = 100$ m.



(d) Uniform capacity linear network for $r = 1000$ m.



(e) Uniform capacity network with topology shown in Fig. 1(a) for $r = 100$ m.



(f) Uniform capacity network with topology shown in Fig. 1(a) for $r = 1000$ m.

Fig. 6. Uniform Capacity for circular and linear networks.

The advantages of multi hop routing strongly depend on the topology: a certain minimum distance between receiving and transmitting nodes and the presence of appropriate relaying nodes are required. If these conditions are satisfied, multi hop routing increases the uniform capacity and additionally decreases the influence of interference since transmission schemes in which an interfering node is nearer to the receiving node than the according transmitter can be avoided. Therefore, the gain of FDMA is decreased if multi hop routing is used.

The influence of noise and interference primarily depends on the inter-node distances. For networks with large differences within these distances the factors and trade-offs influencing the uniform capacity are much more complex, since the possible ratios of signal to interference and noise power cover a wide range. The results suggest that combining FDMA to avoid interference of nearby nodes and multi hop routing to communicate with far nodes is the optimal solution.

Interestingly, the simulations indicate that a static channel assignment can provide a uniform capacity close to the more general case of a dynamic channel assignment, so a fixed but optimal a priori receive channel assignment is sufficient for small networks.

References

[1] S. Toumpis and A. Goldsmith, “Capacity regions for wireless ad hoc networks,” *IEEE Transactions on Wireless Communications*, Jul 2003.