On the Frame Design for Joint OFDM Radar and Communications

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***Abstract*— We discuss how to design OFDM signals which are suitable for both radar and communications using high-bandwidth OFDM signals. A list of constraints which the signal configuration must comply with is defined. We present a solution which satisfies the requirements for radar and allows for reliable communications.**

***Index Terms*— OFDM, radar, signal design, channel coding**

I. Introduction

IN the domain of vehicular technology, a multitude of electronic systems are currently being developed or installed to aid the driver and increase traffic safety. Among these, radar and communications play a special role: in both cases, they have a high range of up to several hundred metres, and they are very similar in nature. Recent trends to use higher frequencies for communications even make it possible to use both systems in the same bands. An obvious idea is thus to fuse both systems and use the same signals and hardware for both radar and communications. This is not only technically feasible, but also economical since less hardware is required and spectrum is used efficiently.

An enabling technology for a combined radar and communications system is OFDM radar as proposed by Sturm et al. [1]. Signals are created by modulating data by means of OFDM. If the bandwidth of the OFDM signals is sufficiently high, the reflection of the OFDM signal can very efficiently be processed to obtain estimates for distance and relative speed of other objects, as explained in [1] and [2]. In [3] we show that the actual data which is modulated onto the signal is not relevant, as long as it is uncorrelated. This makes OFDM a good choice for such a fusion of systems, in particular in the domain of vehicular technology. However, the results should ideally be generally applicable to mobile ad hoc networks.

In order to create a reliable communication link between vehicles and be able to obtain reliable radar information, the details of the system parametrization are important. In this paper, we will list the constraining factors of such a system and propose possible configurations enabling robust communication links and reliable radar imaging.

Several aspects are relevant for the design of such signals. Physical parameters, such as subcarrier spacing, bandwidth and signal duration are as important as the frame structure and channel coding. One major challenge is to define criteria by which a given set of parameters can be validated.

This paper is organised as follows: Section II describes the system and the available parameters. In Section III, the constraints given by the radar and communication systems are listed and defined. Some notes on channel coding are given in Section IV. Finally, Section V concludes.

II. System Model

The entire system featuring both radar and communication is outlined in Figure 1. When transmitting, a receiver is active at the same time to detect any backscattered signals from reflecting objects in the vicinity. The received signals are passed through an OFDM demodulator which runs synchronously to the modulator. A radar processing algorithms takes both transmitted and received signals to estimate distance and Doppler shift of surrounding targets.

When there is no active transmission, the radio system can detect and receive transmissions from other participants, which themselves can use the spectrum to gain a radar image.

The signal is composed of frames. One frame contains one data packet for the communication subsystem and is the signal element which is used for radar imaging.

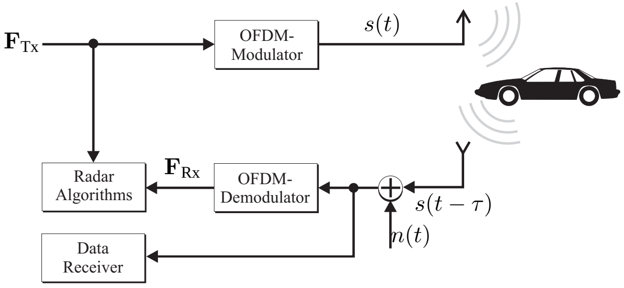

Fig. 1. Schematic of the radar transmitter and receiver

A. The radar subsystem

For a simpler explanation of the radar subsystem, a transmit frame is represented as

$$\mathbf{F}_{\text{Tx}} = \begin{pmatrix} c_{0,0} & \cdots & c_{0,M-1} \\ c_{1,0} & \cdots & c_{1,M-1} \\ \vdots & \ddots & \vdots \\ c_{N-1,0} & \cdots & c_{N-1,M-1} \end{pmatrix}, \quad (1)$$

which denotes a matrix of complex modulation symbols modulated as an OFDM signal; every row of $\mathbf{F}_{\text{Tx}}$ represents one sub-carrier of the OFDM signal, every column represents an OFDM symbol. In parallel to transmitting, an OFDM demodulator using the same local oscillator receives the transmitted frames and passes it to a radar component, which extracts the time delay and Doppler shift between transmitted and received signals according to [1]. Other vehicles can detect and demodulate the signal to receive the transmitted data. This allows the transmission of, e.g., traffic-safety related information to other vehicles as well as the simultaneous radar-imaging of the surroundings.

The received frame can be represented as another matrix $\mathbf{F}_{\text{Rx}}$. Before radar processing, the known elements of $\mathbf{F}_{\text{Rx}}$, the modulated information, is removed from the matrix by element-wise division of $\mathbf{F}_{\text{Tx}}$ and $\mathbf{F}_{\text{Rx}}$, and in the case of a single reflecting object is given by [3]

$$(\mathbf{F})_{k,l} = \frac{(\mathbf{F}_{\text{Rx}})_{k,l}}{(\mathbf{F}_{\text{Tx}})_{k,l}} = e^{j(2\pi(lT_O f_D - k\tau\Delta f)+\varphi)} + (\mathbf{W})_{k,l}. \quad (2)$$

Here, f_D and τ are the Doppler shift and roundtrip propagation time caused by the velocity and distance of the reflecting object. φ is an unknown phase shift. The matrix $\mathbf{W}$ is an $N \times M$ matrix of i.i.d. entries from a complex normal distribution with zero mean and variance σ^2, representing the receiver noise. It must be noted that the simple notation of (2) is only valid if no de-orthogonalisation of the OFDM signal occurs at the receiver [3].

From (2), we can tell that the estimation of f_D and τ is equivalent to the estimation of the frequencies of two orthogonal complex oscillations. A possible method for estimation is the maximum likelihood estimator introduced in [1] and analysed in [3].

B. OFDM parametrization

The relevant physical parameters for the OFDM modulator are the number of carriers N, the number of OFDM symbols per frame M, the sub-carrier spacing Δf and the duration of the guard interval T_G. In addition, we will introduce the option to transmit information on only every U-th carrier, leaving the rest of the sub-carriers empty. The total transmit power is limited to P_t.

III. Constraints

In order to enable reliable functionality of both communications and radar, are large catalogue of constraints must be met. The following sections list several constraints and quantifies them.

A. Channel Constraints

The mobile propagation paths between participants constitute a first set of constraints for the OFDM signal design. In particular, mobile vehicular networks come with challenging propagation channels since both multipath propagation as time-variance introduce fading effects. It must be ensured that the multipath effects do not affect the orthogonality of the signals [4], [5]. The following conditions need to be met:

1) The guard interval length needs to be larger than the *maximum excess delay* τ_e, i.e. the time difference between arrival of the first and last propagation path.

$$T_G > \tau_e$$

2) The carrier distance Δf needs to be smaller than the *coherence bandwidth* B_C, i.e. the frequency span over which the channel can be assumed constant. It must also be much larger than the *Doppler spread* B_D, i.e. the widening of the spectrum as result of the different Doppler shifts, such that the spreading does not destroy orthogonality between carriers.

$$B_D \ll \Delta f < B_C$$

Since OFDM systems are very sensitive to de-orthogonalization, a lower *channel limit* is defined as ten times the Doppler spread.

3) The *coherence time* T_C, i.e. the time over which a channel can be assumed approximately constant, must not exceed the time T_E between channel estimations, at the very least the time of one OFDM symbol.

$$T_E < T_C \ , \ \Delta f^{-1} \ll T_C$$

To quantify these bounds, we needed to acquire information about the channels. For this work, we used a *RayTracer* to create channel data from simulated traffic. This method consists of two steps: first, *traffic scenarios* are generated, including roadside buildings, vegetation and moving vehicles, among which are the transmitting and receiving vehicle. Every vehicle is assigned a trajectory. The position of every vehicle is re-calculated every 10 ms of simulation time, yielding a *snapshot*. In the second step, the wave propagation between two vehicles is calculated using ray tracing methods [6]. This process returns a list of K propagation paths per snapshot, each with Doppler shift, time delay, attenuation and phase rotation. The received signal $r(t)$ can be calculated from this list according to

$$r(t) = \sum_{k=0}^{K-1} \underline{a}_k s(t - \tau_k) e^{j2\pi f_{\mathrm{D},k}(t-\tau_k)} \qquad (3)$$

where $\underline{a}_k$ is a complex attenuation factor, τ_k and $f_{\mathrm{D},k}$ are the delay and Doppler shift of the k-th path, respectively, and $s(t)$ is the transmitted signal.

In total, ten different traffic simulations were created, chosen such that a wide variety of situations was covered. Among these were eight urban scenarios with a variety of traffic and building densities and two highway scenarios, in which vehicles move at high velocities. A maximum relevant communication distance of 100 m was defined, and only snapshots with a distance between transmitter and receiver smaller than that were considered. In total, 10567 snapshots were finally used for analysis, yielding the same number of different channels.

Besides the maximum distance, a power attenuation threshold T_P is introduced. Propagation paths with a power attenuation of more than T_P compared to the strongest propagation path shall not be considered. Throughout this work, T_P shall be fixed at 40 dB [5].

TABLE I

CHANNEL LIMITATIONS FOR THE OFDM PARAMETERS

Property	Urban	Autobahn
RMS excess delay	0.102 µs	0.122 µs
Maximum Doppler spread	7.24 kHz	5.23 kHz
Coherence bandwidth	2246.1 kHz	1269.53 kHz
RMS Coherence time	0.401 ms	0.46 ms

The RayTracing channels were analysed in the following manner: from every snapshot, all propagation paths attenuated beyond the threshold T_P were discarded. From the remaining paths, values for the channel characteristics were derived by the following rules:

- The excess delay τ_e is the time of arrival difference between the first and the last arriving propagation path. From all excess delays, the root mean square (RMS) value was calculated.
- From the Doppler shifts of all propagation paths, the maximum Doppler spread B_D and the RMS Doppler spread $B_{D,\mathrm{RMS}}$ were calculated.
- The RMS coherence time T_C was estimated by the inverse RMS Doppler spread,
$$T_C = \frac{1}{B_{D,\mathrm{RMS}}}.$$
- Path delays were discretized in the time domain at a given sampling rate f_S, such that $\tau_m = k/f_S$ with $k \in \mathbb{N}$. For durations smaller than T_C, the channel is assumed time-invariant, so the Doppler shifts can be ignored and the channel can be represented by its impulse response $h(k)$.
- By taking the z-transform $H_z(z)$ of $h(k)$, the channel frequency response is calculated as $H(f) = H_z(e^{j2\pi f/f_s})$. $\rho_{H,H}(f)$ is the normalized autocorrelation coefficient of the channel frequency response.
- The coherence bandwidth was estimated by the value B_C where the correlation coefficient of the frequency response $\rho_{H,H}(f)$ drops to 90%, i.e.
$$\rho_{H,H}(B_C) = 0.9.$$

Table I gives the results of this analysis.

B. Radar Constraints

As mentioned in Section II, the estimation process assumes there is no error due to de-orthogonalisation. Since the latter is caused by adverse channel effects, the same constraints apply as for the communication system.

More importantly, the accuracy of the radar system is affected by the choice of parametrization. In particular, range and Doppler resolution constrain the bandwidth and frame duration of the signal. For a given minimum range resolution $\Delta d_{\max}$ and a minimum Doppler resolution $\Delta v_{\max}$, the following inequalities need to be met [1], [3]:

$$N\Delta f \geq \frac{c_0}{2\Delta d_{\max}} \tag{4}$$

$$T_F \geq \frac{c_0}{2\Delta v_{\max} f_c} \tag{5}$$

Here, T_F denotes the length of one frame; f_c is the signal's centre frequency.

The choice of the radar processing algorithm also affects the possible parametrizations for the OFDM signals. In [3], we identify a threshold effect for the maximum likelihood estimator, which states that for SNR values below a certain threshold value, the estimates become unreliable. A method to find the SNR value is given in [3].

C. Hardware constraints

In certain cases, the choice of hardware can have effects on the quality of the signal processing. For the given case, high bandwidths are necessary for a high range resolution, and due to the high attenuation of the reflected signals, a high dynamic range must be covered at the radio front-end. In order not to increase the requirements towards the receiver linearity, reducing the peak-to-average power ratio (PAPR) will therefore reduce the non-linear signal distortions introduced by the hardware.

PAPR can be reduced by minimizing the number of sub-carriers. However, as stated by [7], PAPR will not exceed a value of $2\ln N$ with high probability for a large number of carriers; so the issue is more effectively addressed somehow else, e.g. by PAPR-reducing coding methods.

D. Data link constraints

Having met all constraints for radar and channel, a data link quality must also be established. The problem addressed here is the limitation of the total transmit power. Since the bandwidth requirements are very high, only little power is available per sub-carrier. For vehicular applications, a reliable data link is very important. We therefore introduce a minimum frame error rate of 1% which must be met by the design.

Data rate is not considered as important as reliability, and no constraints shall be enforced here. This figure can be used to optimize the setup, i.e. analysing if the data rate can be increased while still holding all constraints.

Parameter	Value
Number of carriers N	1024
Number of symbols M	256
OFDM symbol duration T	11 µs
Guard interval fraction G	$1/8$
Total OFDM Symbol duration T_O	$(1+G)T =$ 12.375 µs
Bits per Modulation Symbol N_b	1 (BPSK)
Sub-carrier spacing U	8

TABLE II
OFDM SYSTEM PARAMETERS

E. A possible configuration

Table II lists a possible configuration which meets all requirements listed in the previous subsections. This configuration also has been tested in live measurements [8] for the radar subsystem, using a centre frequency of 24 GHz. At this frequency, the total transmit power P_t is limited to 100 mW in the EU.

IV. CHANNEL CODING

The choice of the channel code is an essential one in the design of OFDM frames. Its main purpose is to increase the reliability of the communication link by decreasing the bit error rate (BER). One problem for combined radar and communication using OFDM is the high bandwidth: from Table II, the total signal bandwidth is 93.09 MHz. Since the total transmit power is limited, this results in small SNR values on the individual sub-carriers. From a communications point of view, the large bandwidth is therefore more of a hindrance than helpful. The channel code of choice must therefore have good error correction properties.

Many different types of channel codes exist, and of these, there are several codes with the required error correction capabilities. In [9], we suggest Reed-Muller Codes (RM-Codes) as a possible solution, which also have the advantage of limiting the PAPR (or, more precisely, the peak-to-mean-envelope power ratio, PMEPR) as requested in Section III-C. This solution was recently also suggested in [10] for multistatic OFDM radar systems.

The reason for the PMEPR-limiting capabilities of RM-codes is the their connection to Golay sequences, as shown in [11]. In [12], Popovic shows that the PMEPR of any Golay sequence is bounded by 3 dB.

Of particular interest is the code $RM_2(2,m)$, which can be partitioned into cosets of $RM_2(1,m)$. By only using those codewords belonging to Golay cosets, the PMEPR is bounded to 3 dB. Within $RM_2(2,m)$, there are $m!/2$ Golay cosets, each with $2^(m+1)$ codewords. When only using one coset, the code rate drops to $R_c = \frac{m+1}{2^m}$. In this case, the minimum Hamming distance becomes $d_{\text{min}} = 2^{m-1}$. Such a code can easily be maximum likelihood decoded using the fast Hadamard transform.

In the given configuration, where $U = 8$, we employ a single Golay coset with $m = 7$, resulting in one codeword of length 128 which is then mapped to one OFDM symbol each. The achievable data rate of this system is $r = \frac{N}{U} \cdot R_c \cdot \frac{1}{T_O} \approx 0.64\,\text{MBit/s}$, which is not highly spectrally efficient, but reliable. The data rate is enough for traffic safety applications, but can be increased by using more Golay cosets, at the cost of smaller Hamming distance and higher decoding complexity.

Simulations confirm the applicability of RM-codes. In the given channel database, simulations yielded a zero frame error rate for all the urban traffic situations, and a frame error rate of $2.30 \cdot 10^{-3}$ in the highway scenarios [9].

On a side note, the delay caused by the channel coding is the smallest delay possible. However, the coding scheme comes with one inherent disadvantage: since the code words are exactly mapped to OFDM symbols, there is no space left for pilot symbols. This implies that periodically, entire OFDM symbols must be reserved for pilot symbols, which in turn affects the data rate.

V. Conclusion

In this paper, we present an overview of constraining factors for the signal design in joint OFDM radar and communication systems and give a possible configuration which meets all requirements. The radar performance of this setup has also been verified with measurements, see [8]. It should be noted that the large number of degrees of freedom implies a wide variety of possible solutions. The given solution might not be optimal; future research will include further optimization of the signal structure, which will be verified in both simulation and measurements.

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