A Simple Stochastic Channel Simulator for Car-to-Car Communication at 24 GHz

Micha Linde, Communications Engineering Lab (CEL), Karlsruhe Institute of Technology (KIT), Germany
micha.linde@student.kit.edu

***Abstract*— A channel model, based on stochastics and Ray tracing, is described for a communication system, in which moving participants communicate in the 24 GHz band. A realistic behaviour of propagation effects, both short-term and long-term, is modeled. This allows to simulate and design future vehicle-to-vehicle systems.**

I. Introduction

Due to the increasing importance of safety systems in today's automotive technology, many vehicles are able to share safety relevant information. Information such as traffic status, accidents or weather situations are transmitted using mobile communication systems. Furthermore, safety features like adaptive cruise controls (ACC) or brake assist systems use radar techniques. Consequently, both systems can be linked to an integrated communication and radar system and will form a cost and spectrum efficient platform as described in [1].

In order to design, optimise or test specific techniques such as modulation type, channel coding or interleaving for a joint radar and communication system for vehicle-to-vehicle (V2V) applications detailed knowledge of the channel behaviour is required. An exemplary system model including OFDM transmitter, OFDM *(Orthogonal Frequency Division Multiplex)* receiver and signal processing has been implemented [2]. A channel model close to reality is developed and described in this paper.

The transmitted signals reach the receiver across multiple paths. They are affected by scattering, diffraction and reflection as well as the Doppler effect caused by the motion of the transmitter and the receiver or movement of other objects in the immediate vicinity. This multipath propagation results in the time variant and frequency selective behaviour, like delay spread, Doppler spread and fading.

Three methods are known to determine and model these effects.

1. The *measurement based model* is defined by an intensive measurement campaign followed by a detailed analysis. The result is an accurate description of reality but requires cost intensive resources.
2. *Ray tracing* offers the opportunity to model the whole scenario with vectors. Optical techniques are used to create a realistic approach of the multipath propagation. Unfortunately, this method requires enormous computing resources and the results depend heavily on the modeled scenario.
3. The *stochastic methods* models the propagation effects usually by coloured Gaussian processes [3]. This method's outcomes are suboptimal results due to its simplicity and universal validity.

This paper describes a simple stochastic model, which is based on several Ray Tracing models. The paper first introduces known mobile channels, then provides an overview about their typical characteristics and gives a short description of COST 207, a known stochastic model. Section III explains and analyses the discussed scenarios and a simple stochastic model is established, based on COST 207, to model short-term influences of the mobile channel. A method to consider the long-term statistics of the V2V channels follows. Section IV gives an exemplary result and concludes this paper.

II. Description of mobile channels

A. System functions

The following Section is based on [4] and [5]. The channel is assumed to be a linear timevariant filter to describe the time variant and frequency selective behaviour. It is entirely described by the complex time variant channel impulse response (CIR) $h(t,\tau)$, which is defined as the response of the channel at time t to a delta pulse $\delta(t)$ simulating the channel at time $(t-\tau)$.

The CIR is composed of several different transformed copies of the transmitted pulse to simulate the multipath propagation seen in real V2V communication. Each delayed (τ_n) path is subject to a Doppler shift $f_{D,n}$, an attenuation $\alpha_n(t)$ and a phase shift which is described by $\phi_n(t)$; here, n stands for the index of the path. So, the CIR can be described by:

$$h(\tau,t) = \sum_{n=1}^{N(t)} \underbrace{\alpha_n(t)e^{-j2\pi\phi_n(t)}}_{\underline{\alpha}_n(t)} \underbrace{e^{-j2\pi f_{D,n}(t)t}}_{\mu_n(t)} \delta(t-\tau_n(t)) \quad (1)$$

Some paths will erase or amplify each other, because of the coherent sum of copies, with the same delay τ_n but different phase values ϕ_n.

These multipath effects result in a variation of the received signal power, which is known as fast fading. The transversal structure shown in Figure 1 is the most commonly used approach to implement channel models [5].

The transfer function $H(f,t)$, which is given by the Fourier transform of $h(\tau,t)$ w.r.t. τ

$$H(f,t) = \int_{-\infty}^{\infty} h(\tau,t)e^{-j2\pi f\tau} d\tau \quad (2)$$

describes the frequency behaviour of channels.

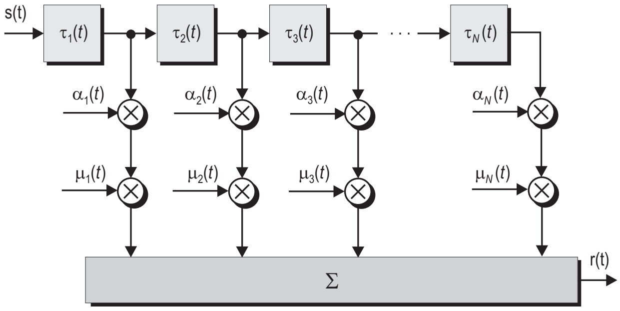


Fig. 1. Commonly used structure to implement channel models

B. WSSUS model

The channel is usually described as a stochastic process, which analyses and describes the stochastic parameters of propagation effects. It can be analysed by the autocorrelation function (ACF) of the transfer function

$$r_{HH}(f_1,t_1;f_2,t_2) = E\{H^*(f_1,t_1)H(f_2,t2)\}\ . \qquad (3)$$

A simplification of (3) was introduced by P.A. Bello [4] and is known as the *Wide-Sense-Stationary Uncorrelated-Scattering* model (WSSUS). A linear superposition of uncorrelated echos and a wide-sense-stationary behaviour of the channel is assumed. With these approximations it can be shown that (3) can be described only by the time $\Delta t = t_2 - t_1$ and the frequency difference $\Delta f = f_2 - f_1$,

$$r_{HH}(f_1,t_1;f_2,t_2) = r_{HH}(\Delta t,\Delta f)\ . \qquad (4)$$

Based on (4) it is possible to specify four characteristic functions. Setting $\Delta t = 0$ results in the *frequency correlation function* $r_{HH}(0,\Delta f)$ describing the frequency selective behaviour. The transform into the frequency domain leads to the *power delay spectrum* $S_\tau(\tau)$. It is proportional to the probability density of the delay values $p(\tau)$. The time variation can be described through the *time correlation function* $r_{HH}(\Delta t, 0)$ and its Fourier transformation, which is called *Doppler spectrum* $S_\mu(f_D)$, and equates to the Doppler probability density $p(f_D)$.

C. Channel characteristics

Based on these four equations, it is possible to define four values, which are statistical parameters of the time variant and frequency selective behaviour of a mobile channel. The frequency behaviour is described by the *delay spread* τ_{DS} and *coherence bandwidth* B_{coh}. τ_{DS} can be specified through the second central moment of $S_\tau(\tau)$

$$\tau_{\mathrm{DS}} = \frac{\int_{-\infty}^{\infty}(\tau-\bar{\tau})^2 S_\tau(\tau)\,d\tau}{\int_{-\infty}^{\infty} S_\tau(\tau)\,d\tau}\ , \qquad (5)$$

where $\bar{\tau}$ describes the mean delay. The coherence bandwidth B_{coh} is defined as the half value period of $r_{HH}(\Delta f, 0)$. Both describe the time dispersive nature and therefore the frequency selective behaviour of the channel.

The analysis of the Doppler spectrum $S_\mu(f)$ shows the time varying characteristics. The Doppler spread B_{DS} is defined similarly to τ_{DS} as

$$B_{\mathrm{DS}} = \frac{\int_{-\infty}^{\infty}(f-\bar{f}_D)^2 S_\mu(f)\,df}{\int_{-\infty}^{\infty} S_\mu(f)\,df}. \qquad (6)$$

This describes the dispersion in frequency domain and the half value period of $r_{HH}(0,\Delta t)$, known as the *coherence time* T_{c}. $\bar{f}_D$ describes the mean Doppler shift.

Therefore, the coherence time T_{c} is a statistical dimension for the time duration in which the channel behaviour is essentially invariant. In analogue, the coherence bandwidth describes the frequency range in which the channel can be assumed to be constant.

The *time-bandwidth product* fundamentally states that [6]

$$T_c \propto \frac{1}{B_{\mathrm{DS}}}\ , \qquad (7)$$

$$B_{\mathrm{coh}} \propto \frac{1}{\tau_{\mathrm{DS}}}\ . \qquad (8)$$

There are also two other empirical values to describe the fast fading behaviour. *Level crossing rate* (LCR), which describes how often the signal falls below a specified signal level R, and the *average fade duration* (AFD), which describes the time seen the signal below this threshold [6].

D. COST 207 model

The COST 207 project, used for simulation of the GSM system, proposes reference models to describe the multipath propagation in four scenarios: rural area (RA), typical urban (TU), bad urban (BU) and hilly terrain (HT). The channel is modeled in terms of a set of delay and Doppler characteristics. The delay power spectrum $S_\tau(\tau)$ is defined as one or two decaying exponential functions, with scenario specific parameters. Four classes of Doppler spectra $S_\mu(f)$ are proposed. These are the Jakes spectrum, two spectra based on Gaussian distributions and the addition of a single direct path to the classic Jakes model, the Rician spectrum. A detailed description can be found in [5].

III. A SIMPLE STOCHASTIC CHANNEL MODEL

We focus on three typical traffic situations. The URBAN scenario describes a typical situation in medium density areas, such as villages or small cities. The BAD URBAN scenario demonstrates worse situations, in which the density is very high and a lot of traffic is expected. The third case, AUTOBAHN, describes typical situations on highways or country roads in rural areas with less traffic. Several reference channels were simulated by Raytracing [3] to develop a stochastic model for these situations. All analysed simulations have one similar feature: the channel behaviour can split up into its short-term statistic, which is in particular responsible for delay and Doppler spread, and a long-term statistic having significant impact on fading behaviour. Figure 4 exemplifies a section of the correlation coefficients $r_{\tau_n}(t;t+1)$ of the delay vector τ_n between adjacent channels. There are some sections which act with a explicit higher correlation (e.g. between $t = 0.825$ s and $t = 0.86$ s) because the channel characteristics change insignificantly.

Table I lists the relevant parameters, determined by the three simulated Raytracer scenarios.

TABLE I
CHARACTERISTICS OF V2V CHANNELS

Parameter	URBAN	BAD URBAN	AUTOBAHN
$\bar{\tau}_{DS,rms}$	0.38 μs	0.36 μs	0.23 μs
$\tau_{DS,max}$	2.50 μs	3.11 μs	1.7 μs
$\bar{B}_{DS,rms}$	3.39 kHz	4.48 kHz	4.2 kHz
$B_{DS,max}$	7.20 kHz	8.76 kHz	13.55 kHz
$\bar{T}_{coh}$	0.24 ms	0.21 ms	0.22 ms
$\bar{B}_{coh}$	2.78 MHz	2.58 MHz	4.02 MHz
LCR	17.68 1/s	19.22 1/s	17.27 1/s
AFD	36.04 ms	32.01 ms	25.22 ms

A. Modeling short-term statistics

We assume a modified COST 207 model, which is adapted to the special cases of the V2V channel at 24 GHz. The COST project proposes reference models for the Doppler spectrum $S_\mu(f)$ and the delay power density spectrum $S_\tau(\tau)$ for different scenarios.

1) Delay power spectrum: The delay power spectrum $S_\tau(\tau)$ is modeled by one or two decaying exponential functions

$$S_\tau(\tau) = c \cdot e^{-\beta(\tau_s - \tau)} \quad \text{for} \quad \tau_s \leq \tau < \tau_D \tag{9}$$

in which β describes the gradient, τ_s and τ_D the start and stop time and c the weighting of the exponential function. N_{exp} stands for the number of exponential functions. Several raytracer simulations of the proposed situations offer the possibility to get a simple approximation of the required parameters by using the numerical Gauss-Newton method. Figure 2(a) - 2(c) show the superposition of several delay profiles, which are simulated by raytracing. Also, the approximation $S_\tau(\tau)$ is shown.

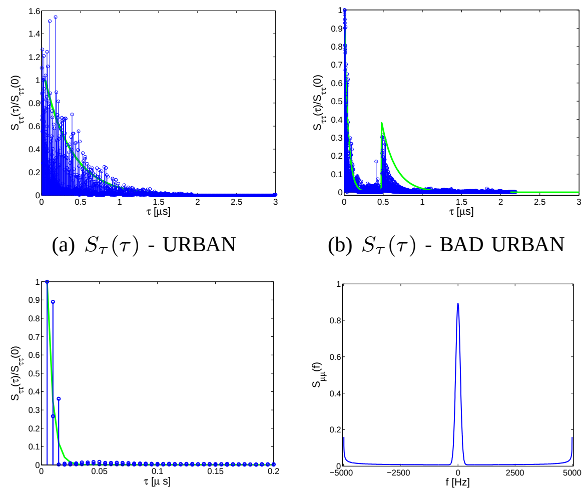
2) Doppler power spectrum: The Doppler behaviour is modelled by a typical Jakes spectrum, which is superimposed with the direct components from the LOS path. The results of the raytracer show that there is not only one dominating component. The Rician peak is widened to a Gaussian function with variance σ_{rice} and the average f_{rice} which is equivalent to the Doppler value of the direct component. Therefore, the Doppler power spectrum is given by

$$S_\mu(f) = e^{\frac{(f - f_{\text{rice}})^2}{2\sigma_{\text{rice}}{}^2}} \tag{10}$$

for paths with small delay ($0 \leq \tau < \tau_{\text{rice}}$) and

$$S_\mu(f) = \frac{1}{\pi f_{\max}\sqrt{1 - (f/f_{\max})^2}} \tag{11}$$

for the Jakes components ($\tau \geq \tau_{\text{rice}}$) (Figure 2(d)). Here, $f_{\max}$ stands for the maximum possible Doppler shift. Table II lists the parameters, determined by several Raytracer simulations.



(a) $S_\tau(\tau)$ - URBAN
(b) $S_\tau(\tau)$ - BAD URBAN
(c) $S_\tau(\tau)$ - AUTOBAHN
(d) $S_\mu(f)$

Fig. 2. Approximation of $S_\tau(\tau)$ and $S_\mu(f)$

TABLE II
PARAMETRIZATION OF $S_\tau(\tau)$ AND $S_\mu(\mu)$

Parameter	URBAN	AUTOBAHN	BAD URBAN
N_{exp}	1	2	2
β	2.831	[220 ; 25.5]	[20.36 ; 10.2]
c	1	[0.8 ; 0.2]	[0.7 ; 0.3]
τ_s [μs]	0	[0 ; 1.6]	[0 ; 0.49]
τ_D [μs]	2.505	[0.1 ; 1.665]	[0.48 ; 3.108]
f_{rice} [Hz]	-500	-100	0
σ_{rice} [Hz]	300	150	750
τ_{rice} [μs]	0.35	1.6	0.15

3) Structure of the simulator: First, a random number of paths are selected, following a normal distribution $N_{\text{paths}} = N(\sigma^2, \mu^2)$ with scenario typical values for variance and mean. The delay vector τ_n and the Doppler vector μ_n follow the specified distribution $p(\tau)$, which is given by normalised power spectrum's $S_\tau(\tau)$ and $S_\mu(f)$, respectively.

It is possible to generate random numbers $\nu_n = \tau_n$ with the specified distributions $p(\nu)$ by calculating a functional transformation [7]

$$\nu_n = g_\nu(u_n) = P_\nu^{-1}(u_n) \quad \text{for} \quad 0 \leq u_n < 1 \tag{12}$$

where g_ν is the inverse of the cumulative distribution function P_ν (CDF).

As an example we derive the random delay vector τ_n, which is modeled by (9). Therefore, the probability density of τ_n is described by

$$p_\tau = c \cdot S_\tau(\tau) = c \cdot e^{-\beta\tau_n} \quad \text{for } 0 \leq \tau < \tau_D \tag{13}$$

where $c = \beta/(1 - e^{-\beta\tau_D})$ is the scaling factor. For simplification $\tau_s = 0$ is chosen. Then, the cumulative distribution is given by

$$P_{\tau_n}(\tau_n) = \begin{cases} 0 & \text{for } \tau_n < 0, \\ \frac{1-e^{-\beta\tau_n}}{1-e^{-\beta\tau_D}} & \text{for } 0 \leq \tau_n < \tau_D, \\ 1 & \text{else} \end{cases} \tag{14}$$

and application of (12) results in

$$\tau_n = -\frac{1}{\beta}\ln(1-u_n[1-e^{-\beta\tau_D}]) \quad \text{for} \quad 0 \le u_n < 1 \ . \qquad (15)$$

Equivalent expressions can be given for several clusters and the Doppler vector μ_n.

The phase vector is uniformly distributed according to the WSSUS assumption and the complex attenuation value $\bar{\alpha}_n$ of the n-th path is distributed according to a complex Gaussian distribution. $\bar{\alpha}_n$ is weighted with the associated value of $S_\tau(\tau_n)$ to consider the path loss.

B. Modeling long-term statistics

It makes sense to control the generation inverse $\tau_n(t)$ and $\mu_n(t)$ by models, which can be considered as very simple Markov-models [8] as shown in Figure 3. In this way, the described correlation between adjacent channels can be created to get a more realistic model of the channel behaviour and in particular the fading characteristics. The state Z1 symbolises a not significant alteration about the time, hence $\tau_n(t) = \tau_n(t-1)$ is chosen and in contrast to this, Z0 stands for a random regeneration of the vector. The transition probabilities $p(Z_i; Z_j)$ are defined by analysing and comparing to a threshold $\rho = 1 - 1/e$ of the correlation $r_x(t; t+1)$ between the input parameters of the raytracer (Figure 4 and Table III).

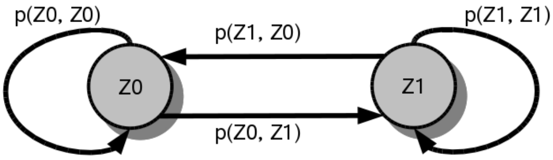


Fig. 3. Markov-Model

1) Generation of correlated Rayleigh fading: A procedure for generating $\alpha_n(t)$ is needed to generate an accurate model of the fading behaviour. Its envelope has to follow a desired cross-correlation, given by $\rho = \mathcal{N}(\bar{r}_{|\alpha|}(t;t+1), \sigma^2_{r_{|\alpha|}(t;t+1)})$, but the phase vector must be uncorrelated according to the WSSUS condition. The following section follows [9].

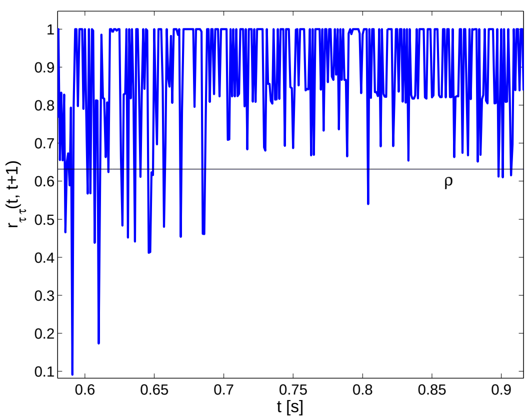


Fig. 4. Correlation coefficient of τ_n $(r_\tau(t, t+1))$

TABLE III
TRANSITION PROPAPILITIES $p(z_i, z_j)$ FOR τ_n AND CORRELATION COEFFICIENTS

i, j	$z0, z0$	$z0, z1$	$z1, z0$	$z1, z1$	$\bar{r}_\tau$	σ_{r_τ}
URBAN	0.22	0.78	0.11	0.89	0.55	0.27
BAD URBAN	0.42	0.58	0.14	0.86	0.48	0.21
AUTOBAHN	0.19	0.81	0.03	0.97	0.72	0.35

$\alpha_n(t_1)$ and $\alpha_n(t_2)$ are complex Gaussian samples, which can be expressed through their real and imaginary components

$$\alpha_n(t_1) = a_n(t_1) + j \cdot b_n(t_1) \qquad (16)$$

$$\alpha_n(t_2) = a_n(t_2) + j \cdot b_n(t_2) \ . \qquad (17)$$

The fading behaviour is only characterised by their envelopes

$$r_n(t_1) = |\alpha_n(t_1)| = \sqrt{a_n(t_1)^2 + b_n(t_1)^2} \qquad (18)$$

$$r_n(t_2) = |\alpha_n(t_2)| = \sqrt{a_n(t_2)^2 + b_n(t_2)^2} \ , \qquad (19)$$

which follow the known Rayleigh distribution. With a view to generate two Rayleigh sequences $r_n(t_1)$ and $r_n(t_2)$ with equal power, the correlations values between the real and imaginary components of $\alpha_n(t_1)$ and $\alpha_n(t_1)$ are:

$$\begin{aligned} E\{a_n(t_1)^2\} &= E\{a_n(t_2)^2\} = E\{b_n(t_1)^2\} \\ &= E\{b_n(t_2)^2\} = u \qquad (20) \end{aligned}$$

$$E\{a_n(t_1)b_n(t_1)\} = E\{a_n(t_2)b_n(t_2)\} = 0 \qquad (21)$$

$$E\{a_n(t_1)a_n(t_2)\} = E\{b_n(t_1)b_n(t_2)\} = u_1 \qquad (22)$$

$$E\{a_n(t_1)b_n(t_2)\} = -E\{b_n(t_1)a_n(t_2)\} = u_2 \ . \qquad (23)$$

The normalized cross-correlation between the envelopes $r_n(t_1)$ and $r_n(t_2)$ are [10]

$$\rho_{\text{CCF}} = \frac{\frac{\pi}{2}F(-\frac{1}{2}, -\frac{1}{2}; 1; \lambda^2) - \frac{\pi}{2}}{2 - \frac{\pi}{2}} \ , \qquad (24)$$

where

$$\lambda^2 = \frac{u_1^2 + u_2^2}{u^2} \qquad (25)$$

symbolises the squared magnitude of the cross-correlation coefficients. $F(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric function. We determine the correlation properties of the complex Gaussian random variables, which are defined through λ, to generate two envelopes with a specified cross-correlation ρ_{CCF}. It is impossible to solve (24) in closed form, but a good approximation can be found by applying a root-finding algorithm[9].

The correlation between two complex Gaussian samples $\alpha = [\alpha_n(t_1)\ \alpha_n(t_2)]^T$ can be given by its correlation matrix

$$R_{\alpha\alpha} = E\{\alpha\alpha^H\} = \begin{bmatrix} 2u & 2u_1 - j2u_2 \\ 2u_1 + j2u_2 & 2u \end{bmatrix} , \qquad (26)$$

where $2u$ is equivalent to the signal power σ^2_α of α. With (25) and the simplification $u_1 = u_2$ (26) can be simplified to

$$R_{\alpha\alpha} = \begin{bmatrix} \sigma^2_\alpha & \frac{1}{\sqrt{(2)}}\lambda\sigma^2_\alpha(1-j) \\ \frac{1}{\sqrt{(2)}}\lambda\sigma^2_\alpha(1-j) & \sigma^2_\alpha \end{bmatrix} . \qquad (27)$$

(27) describes the correlation matrix of two Gaussian sequences, whose envelopes follow the specific cross correlation.

We assume two unit power uncorrelated complex Gaussian sequences $w_n(t_1)$ and $w_n(t_2)$ to generate complex Gaussian samples. The correlation matrix for $W = [w_n(t_1)\ w_n(t_2)]^T$ is a two dimensional identity matrix $R_{WW} = I_2$. It is shown in [9], that $R_{\alpha\alpha}$ can be "stamped" on W by

$$\alpha = LW \ , \tag{28}$$

where L is a colouring matrix given by Cholesky decomposition of $R_{\alpha\alpha}$ to

$$L = \begin{bmatrix} \sigma_\alpha & 0 \\ \frac{1}{\sqrt{2}}\lambda\sigma_\alpha(1+j) & \sigma_\alpha\sqrt{1-\lambda^2} \end{bmatrix} . \tag{29}$$

Because of (28), the components of α are weighted sums of the Gaussian components of W, and α is still following a Gaussian distribution as needed. Two envelopes with a high correlation between $\alpha_n(t_1)$ and $\alpha_n(t_2)$ are shown in Figure 5.

IV. Conclusion

In this paper a simple stochastic channel model for typical V2V communication scenarios based on COST 207 was presented. Several Raytracer simulations were arranged and analysed. Furthermore, a method to model the long-term behaviour of propagation effects has been proposed. An example of a simulated timevariant CIR $h(\tau, t)$ for critical situations with a lot of traffic in typical city's (BAD URBAN) is shown in Figure 6. A verification by intensive measurement campaign has to follow, but typical characteristics delay spread, Doppler spread, coherence time and coherence bandwidth of the stochastic model average the simulated values. Consequently, this channel model permits elementary research of communication techniques, such as modulation type, channel coding or interleaving for a joint radar and communication system at 24 GHz.

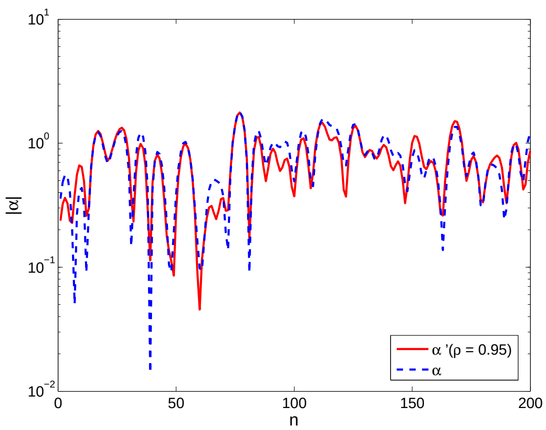


Fig. 5. Two envelopes with high correlation ($\rho = 0.95$)

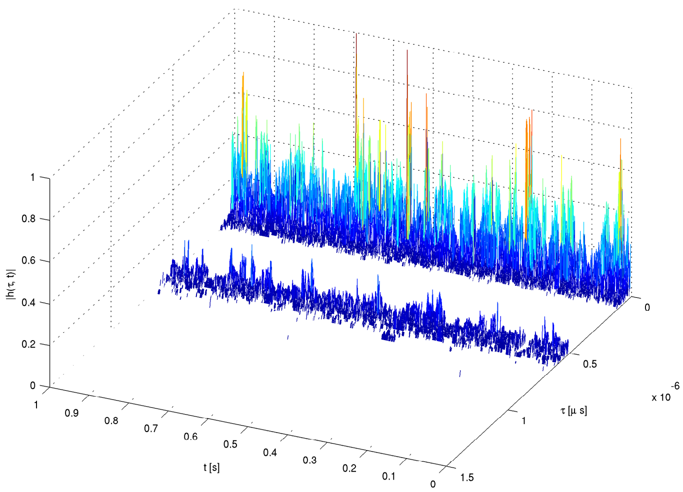


Fig. 6. CIR "BAD URBAN"

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