Short Paper: SIC-MMSE Detection for Filter Bank Multicarrier Systems

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***Abstract*—Filter bank multicarrier is one of the candidates for next generation mobile radio systems. Using filtered sub-channels reduces side-lopes as well as interference among sub-channels. Symbols overlap in time and are not separated by a cyclic prefix resulting in inter-symbol inference as well as interference from adjacent sub-channels under multi-path propagation. Equalization methods known from OFDM can not be directly applied. However, a linear per-subchannel equalizer with minimum mean-squared error has been proposed. This has also been used for iterative decoding and equalization by remodulating decoded symbols to cancel-out interference. In this short paper, we extend this concept and derive a MMSE equalizer using soft interference cancellation. Using soft-information increases performance and enables turbo equalization for filter bank multicarrier systems.**

***Index Terms*—FBMC, SIC-MMSE, Turbo**

I. Introduction

For future mobile radio systems alternative multicarrier schemes are considered in current research. Among the candidates is filter bank multicarrier (FBMC): It uses overlapping filtered sub-carriers which provides good sub-channel isolation and low out-of-band emissions. However, because symbols overlap in both time and frequency, transmissions in multi-path environments are not only affected by ISI but also ICI, due to the loss of orthogonality of directly adjacent sub-carriers.

To recover the transmitted symbols at the receiver a minimum mean-squared error (MMSE) equalizer is derived in [1] which considers not only neighboring symbols on the same sub-carrier but also those of the adjoining ones. This solution is the basis of various extensions trying estimate and cancel-out the ICI in an iterative process of equalizing, hard decoding and remodulation [2], [3].

In this short paper, we propose an MMSE-based linear equalization using a priori information. This extends these concepts and opens the door for iterative decoding using soft-interference cancellation. In turbo equalization [4] the equalizer and the decoder take turns processing soft-information from the other component in an iterative manner. The decoder is agnostic to the underlying waveform, whereas the equalizer uses observations of the receiver filter bank output and the information provided by the decoder as input and, therefore, has to adopted for FBMC systems.

In section II we outline our system model and the derivation of the equalizer coefficients. Some preliminary results based on simulations are shown in section III. Section IV concludes and discusses next steps.

II. Equalizer coefficients

For our system model we use an FBMC-based transmitter and receiver with M sub-channels. The channel is modeled using a time-invariant finite impulse response filter h as well as additive white Gaussian noise $\eta[n]$ with a variance of σ_η^2. We calculate the overall impulse response between a from sub-channel i to sub-channel k by combining the effects of the transmit and receive filter as well as the channel impulse response followed by a down sampling with a factor of $M/2$: $q_{k,i}[n] = (g_i * h * g_k)[m]_{\downarrow \frac{M}{2}}$. Assuming filters with a high stop-band attenuation, only directly neighboring sub-channels contribute to the inference for each symbol: $q_{k,i}[n] \approx 0$ for $|i-k| > 1$. Stacking the real and imaginary part of N observations $y_{k,n}$ of the receive filterbank output samples the following matrix notation can be found

$$\mathbf{y}_{k,n} = \sum_{i=k-1}^{k+1} \mathbf{Q}_{k,i}\mathbf{x}_{k,n} + \mathbf{B}_k \eta[n\tfrac{M}{2}], \tag{1}$$

where $\mathbf{x}_{k,n}$ are transmit symbols from the real-valued alphabet $\{\alpha_j\}_{j=0}^{A-1}$ on sub-channel k contributing the N observations $\mathbf{y}_{k,n} = [y_{k,n-N_2}\ y_{k,n-N_2+1}\ \cdots\ y_{k,n+N_1}]^T$, $N = N_1 + N_2$, via the convolution matrix $\mathbf{Q}_{k,i}$ derived from $q_{k,i}[n]$. The convolution matrix $\mathbf{B}_k$ takes the noise coloring due to the receive filter into account. For a detailed description of this model see [2].

Using analogues derivations as shown in [4] an MMSE-optimal solution for the symbol estimate $\hat{x}_{k,n}$ using the system model (1) can be derived. It is given by

$$\hat{x}_{k,n} = \bar{x}_{k,n} + v_{k,n}\mathbf{f}_{k,n}^T \mathbf{y}_{k,n}^0, \tag{2}$$

where

$$\mathbf{y}_{k,n}^0 = \mathbf{y}_{k,n} - \sum_{i=k-1}^{k+1} \mathbf{Q}_{k,i}\bar{\mathbf{x}}_{k,n} \tag{3}$$

are the observations after soft-cancellation. Using $\mathbf{s}_k = \mathbf{e}_{k,\triangle}^T \mathbf{Q}_{k,k}^T$ we get the equalizer impulse response

$$\mathbf{f}_{k,n} = \mathbf{s}_k \left(\sum_{i=k-1}^{k+1} \mathbf{Q}_{k,i}\mathbf{V}_{i,n}\mathbf{Q}_{k,i}^T + \frac{\sigma_\eta^2}{2}\mathbf{B}_k\mathbf{B}_k^T \right)^{-1}. \tag{4}$$

The a-priori information about the transmitted symbols given as log-likelihood ratios (LLR) of the respective binary data they contain. Here, they are represented by their mean $\bar{x}_{k,n}$ and variance $v_{k,n}$, which is also used in the covariance matrix

$\mathbf{V}_{k,n}$. Note, in the context of turbo equalization the desired output log-likelihood ratios represent only the extrinsic part, which means for each output only the a-priori information on the surrounding symbols is used for calculating the estimates $\hat{x}^e_{k,n}$.

As shown in [5] the a-posteriori distribution $p(\hat{x}^e_{k,n}|x_{k,n} = \alpha_i)$ of the symbol estimates can be approximated by a Gaussian distribution with negligible performance degradation. This way only their mean and variance are required:

$$\mu_{k,n,j} = K_{k,n}\alpha_j \mathbf{f}^T_{k,n}\mathbf{s}_k \quad (5)$$

$$\sigma^2_{k,n} = K^2_{k,n}\mathbf{f}^T_{k,n}\mathbf{s}_k\left(1 - v_{k,n}\mathbf{s}^T_k\mathbf{f}_{k,n}\right), \quad (6)$$

with $K_{k,n} = (2+(1-2v_{k,n})\mathbf{s}^T_k\mathbf{f}_{k,n})^{-1}$. If, for example, QPSK symbols are used, each (double-rate) real-valued symbol in our model is from $\{\sqrt{2}, -\sqrt{2}\}$. For this case, the extrinsic LLRs are given by

$$L^e_{k,n} = \frac{|\hat{x}^e_{k,n} - \mu_{k,n,1}|^2 - |\hat{x}^e_{k,n} - \mu_{k,n,0}|^2}{\sigma^2_{k,n}}, \quad (7)$$

in which we put (5) and (6). The resulting equalizer output can be calculated based on the observation $\mathbf{y}_{k,n}$ and the a-priori information $L_{k,n}$ via $\bar{x}_{k,n} = \sqrt{2}\tanh(L_{k,n}/2)$ and $v_{k,n} = 1/2 - |\bar{x}_{k,n}|^2$:

$$L^e_{k,n} = \frac{4\sqrt{2}}{1 - v_{k,n}\mathbf{s}^T_k\mathbf{f}_{k,n}}\mathbf{f}^T_{k,n}\left(\mathbf{y}^0_{k,n} + \mathbf{s}_k\bar{x}_{k,n}\right). \quad (8)$$

This completes the FBMC-specific part of a turbo equalizer. The other required components, namely the interleavers and the channel decoder are for the most part waveform agnostic.

III. Simulations

For our simulations we assume an FBMC system with $M = 128$ OQAM modulated sub-channels using a Mirabbasi-Martin filter [6] and symbols from a QPSK alphabet. We use the same time-invariant channel impulse response for all results. It has four discrete paths with an exponentially decreasing power. The maximum channel delay is at 10% of the symbol duration.

For now, we are only interested in the performance of the equalizer part of our turbo equalizer enabled receiver. That's why we omit the interleavers and the decoder and use random LLRs based on a Normal distribution (see [7] for details). To visualize the impact of having a-priori information we currently use the uncoded bit error rate based from the output LLRs of the equalizer. Of course, this is not really ideal since gradual improvements in the a-posteriori information can not be observed, even though in the full system the decoder would benefit. Further analysis using full turbo equalizer with a typical state-of-the-art decoder and interleavers or EXIT-Charts will be necessary.

Fig. 1 shows the uncoded bit error rate (BER) for different normalized signal-to-noise ratios (E_b/N_0). As parameter we use the mutual information between the symbol source at the transmitter and the input LLRs as parameter to have an easy link to the quality of the a-priori information at the receiver input. No a-priori information is the conventional MMSE equalizer. It can be observed that even though the ISI/ICI is pretty strong for this channel the more a-priori information the equalizer is provided with be better the estimation.

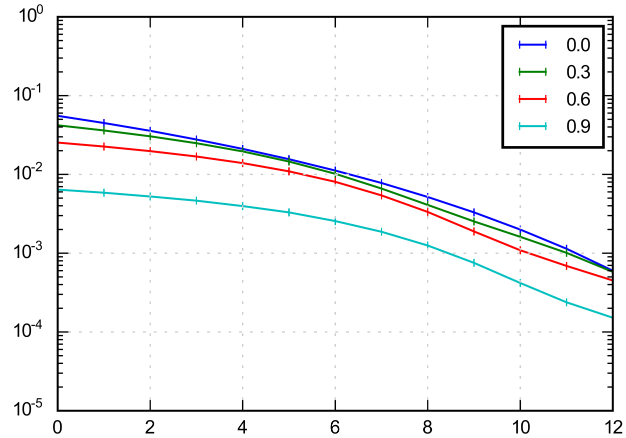

Fig. 1. Performance results for different levels of mean a-priori information

IV. Conclusion

252.0ptIn this short paper we have outlined the steps towards a turbo equalizer enabled FBMC receiver. Preliminary results show the concept is working and that using soft-information enabled us to cancel out some of the ICI.

Next steps are to do a detailed performance analysis of the equalizer and also compare it to other solutions prosed for FBMC. Also, since the computation cost is very high, we will investigate the applicability of sub-optimal algorithms proposed for other transmission systems. Truncation of the overall impulse response as well as time-averaging of the equalizer filter coefficients are promising methods to reduce the computational cost.

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