On the Applicability of the Residual Weighting Algorithm for TDOA

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***Abstract*— Locating a mobile transmitter passively is a challenging task. Recently, the most preferred method for this matter is the Time Difference of Arrival (TDOA) method. In this paper, different techniques for enhancing the TDOA results are introduced. They generally give an idea about the challenges of TDOA. We analyze different ways of using consecutive measurements as well as redundant sensors. We do that focusing on one specific algorithm using TDOA measurements, the residual weighting algorithm (RWA). Its main advantages lie in the low computational complexity and the fact that it doesn't require additional a priori information. We analyze the performance of the algorithm through simulation of different scenarios and present the most preferable application. We show what kind of additional averaging could be used to enhance the results.**

I. INTRODUCTION

Estimating the position of a transmitter is an important issue especially for mobile vendors as well as for frequency regulators. It has gained importance lately because of the increased interest in localizing a mobile user for emergency and security cases. While many techniques have been introduced througout the past two decades, Time Difference of Arrival is considered the most important method when trying to locate a transmitter passively and with as little a priori information as possible. This method only requires a synchronization of the sensors among each other and doesn't require any additional information from the mobile entity. Methods for solving the TDOA problem are based on solving a least squares estimation problem of the non linear equation system of the TDOA geometry. They are basically either closed form solutions or iterative methods. Examples are [1], [2], [3], and [4]. Their main trade off lies between the computational complexity and the estimation accuracy. In highly erroneous environments, additional averaging techniques are neccessary to obtain accurate results.

While least squares estimation is sufficient for zero mean measurement errors, it results in biased solutions in case of a biased error. As Non Line of Sight (NLOS) propagation is expected in mobile channels, the need for dealing with biased errors in TDOA is obvious. Many methods have been developed for this matter [5], [6], [7]. Meanwhile, Kalman Filters are mostly used as an estimation method for TDOA [8],[9], mainly because of their robustness and high accuracy. In this paper, we focus on one method, the residual weighting algorithm introduced by Chen in [10]. The main advantage of this method is the fact that it doesn't depend on a priori knowledge of the NLOS situation. We use this algorithm to present the different aspects and challenges of TDOA in general. We do that through analyzing many implementation aspects of this algorithm. On the one hand we present the constraints and bounds of this algorithm and on the other hand we give solutions and tips on how to optimize the algorithm for a given scenario.

The paper is organized as follows. Section II gives a brief introduction to the TDOA problem and highlights the difference between the zero mean error case and the biased error case. Section III describes Chen's algorithm and introduces different techniques on how to enhance the position estimation using this algorithm. Section IV describes the simulated scenarios and shows the results of the different extra techniques that were implemented in the algorithm. Section V draws the conclusion and gives ideas for further work.

II. THE TDOA LOCATION ESTIMATION PROBLEM

Figure 1 shows a scenario of a TDOA sensor network geometry with three receivers $R_{x_1}, R_{x_2}, R_{x_3}$ and one transmitter T_x. For each pair of receivers, the TDOA between the receivers is estimated in a first step. This step is not considered here and is usually done by cross correlating the synchronized signals of both receivers. The result is a time difference that can be expressed as a distance difference $d_{i,j}$ if the propagation speed c is known:

$$d_{i,j} = c \cdot TDOA_{ij} = d_i - d_j \tag{1}$$

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{2}$$

Equation (2) calculates the distances between the source and the sensors for the two-dimensional case. Inserting equation (1) in equation (2) results in the TDOA equation system given in equation (3). The system could be easily extended for the three-dimensional case.

To reduce computation, one of the N sensors is chosen to be the reference sensor and only $N-1$ TDOAs are estimated with reference to this sensor, resulting in a set of $N-1$ equations given in (3) with two unknown values $[x, y]$. In an error-free environment, the TDOA equations yield a set of hyperbolas that intersect in one point, namely the transmitter position (Fig.1). When error occurs, the hyperbolas shift and can intersect in different points, creating ambiguity (Fig.2). If the error can be modeled as additive white Gaussian noise, then a least squares estimator (LSE) would deliver the optimal

978-1-4673-2017-7/12/$31.00

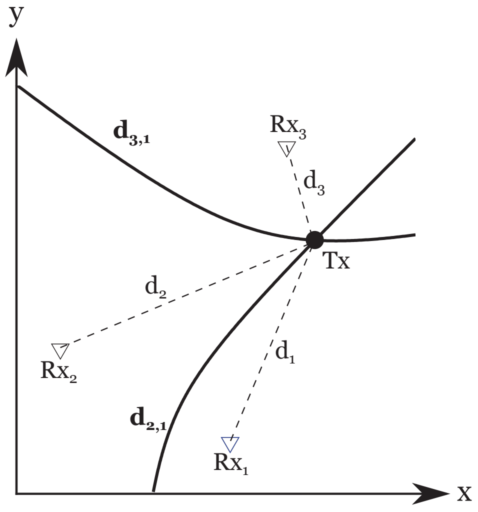

Fig. 1. TDOA-Geometry

result. For biased error sources, this solution would not yield accurate results and can not be improved by averaging.

$$d_{i,j} = \sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_j - x)^2 + (y_j - y)^2} \tag{3}$$

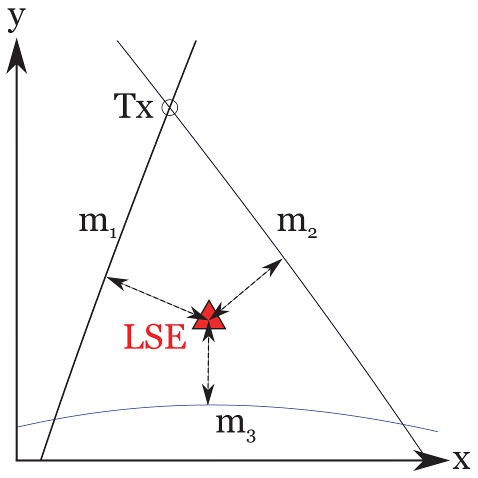

Fig. 2. Erroneous TDOA hyperbolas

III. Extension of Chen's algorithm for TDOA

In a two-dimensional case, the minimum number of sensors required to solve the TDOA equation is 3. Additional measurements can be used to improve the results. Chen [10] does that by arranging the sensors into different subsets containing at least three sensors for the two-dimensional case. Each of these subsets can therefore yield an estimate for the transmitter location using a least squares estimator. For example, 5 sensors contain 10 subsets using 3 sensors, 5 subsets using 4 sensors and 1 subset using 5 sensors. Let S_q denote the receivers of a subset q. Chen's method combines the least squares results of the different subsets using their residuals. A residual is a measure of the deviation of an estimated value from the measured value. In the case of Time Difference of Arrival measurements, the residual of a subset q is calculated by:

$$R_q = \frac{\sum\limits_{i \in S_q, i \neq j} (d_{i,j} - \hat{d}_{i,j})^2}{|S_q| - 1} \tag{4}$$

with $d_{i,j}$ denoting the measured time difference according to the cross correlation of the received signal pairs and $\hat{d}_{i,j}$ being the estimated time difference, i.e.,

$$\hat{d}_{i,j} = \sqrt{(x_i - x_q)^2 + (y_i - y_q)^2} - \sqrt{(x_j - x_q)^2 + (y_j - y_q)^2} \tag{5}$$

with (x_q, y_q) denoting the estimated transmitter position calculated using the measurements of the receivers in subset q.

The Residuum Weighted Algorithm (RWA) method weights the subsets based on the following statement: subsets containing NLOS receivers result, on average, in larger residuals. This can be explained through analyzing the hyperbola shifts according to the different errors. The additive noise results in a random shift of the hyperbola, that leads, on average, to shifts around zero due to the zero mean character of the noise, whereas the NLOS error leads to constant shifts that are proportional to the NLOS bias. So, if we compare the intersection of three hyperbolas containing only zero mean noise with three hyperbolas containing one biased hyperbola, then the probability is higher, that the residual of the subset containing NLOS receivers would be larger. For this reason, the RWA weights the least squares estimates of each subset by the inverse of their residuals. The final estimate is the weighted sum divided by the sum of weights (eq. 6).

$$\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \frac{\sum\limits_{q \in Q_M^L} \frac{1}{R_q} \cdot \begin{pmatrix} x_q \\ y_q \end{pmatrix}}{\sum\limits_{q \in Q_M^L} \frac{1}{R_q}} \tag{6}$$

Q_M^L is the number of subsets that were used to calculate the end result. It has the minimum subset size M and the maximum subset size L. If we have N receivers, then we have a maximum of Q_3^N receivers.

In this paper we investigate the following aspects:

- Which subset sizes should be used to calculate the final result and which subset sizes are unneccessary?
- How can we enhance the result if we have consecutive TDOA measurements? What are the possible techniques? Which one is best?
- Under which circumstances does this algorithm work?

Concerning the first point: In the two-dimensional case, we need at least three TDOA measurements to get an intersection point of the two hyperbolas. Two hyperbolas would either not intersect at all or would intersect in one point. In some cases the hyperbolas intersect in two points, out of which one is usually a clearly wrong solution. In all those cases, a computation of the residual wouldn't be possible and therefore it wouldn't be a measure of the estimation quality. Hence, a minimum number of four receivers is needed for each subset. On the other hand, using the full set with the maximum subset size could only result in biased estimates in case of biased errors.

The second point we analyzed was about how to ideally use K consecutive TDOA measurements $\hat{\tau}_{i,j}$. For that, we tested the three following possibilities:

- TDOA Average: Calculate the average of the K Time Difference measurements $\bar{\tau}_{i,j}$ using equation (7) and use the average to estimate the transmitter position. This technique has the additional advantage of reduced computation, because the location estimation algorithm is executed only once every K-measurements. The disadvantage would be the reduced update rate of the transmitter position.

$$\bar{\tau}_{i,j} = \frac{1}{K} \sum_{k=1}^{K} \hat{\tau}_{i,j}(k) \tag{7}$$

- Subset Average: Calculate K position estimations $\hat{x}_q$ for each subset and build the average of the estimated positions for each subset, then calculate the final result using the RWA.

$$\bar{x}_q = \frac{1}{K} \sum_{k=1}^{K} \hat{x}_q(k) \tag{8}$$

- End Average: Calculate the position estimates using Chen's algorithm in each of the received measurements, then estimate the final result through the mean of the K position estimates.

$$\bar{x} = \frac{1}{K} \sum_{k=1}^{K} \hat{x}(k) \tag{9}$$

Concerning the third question about the constraints of the algorithm, many scenarios with different geometries and NLOS relations were simulated and will be presented in the next section.

For the least squares estimation we have used the iterative Taylor Series Estimation (TSE) method described in [4]. The method starts with an initial guess and iteratively searches for the minimum of the cost function through linearizing the non linear function around the guess. The initial guess was calculated using the centroid of the receiver setup, and for the use of the subset method, the TSE was calculated once using the full set of receivers and the result was used as the initial guess for all the subsets.

The next section shows results of the different averaging techniques in order to compare them to each other. In addition to the different techniques, the results of the estimation using only Line of Sight receivers are also plotted. This can be seen as a comparison to methods that deal with detecting NLOS receivers and eliminating them [5]. These methods have two main drawbacks. On the one hand, they use less sensors for calculating the position estimate, which results in large errors in bad geometry setups. The other problem lies in the assumption that there is one known receiver which has Line of Sight, which is then used as the reference receiver. Therefore, these methods can always assume a positive NLOS measurement error.

IV. Simulation Results

In the simulation, the generated TDOA measurements were corrupted by zero mean Gaussian noise with a given standard deviation (stdv) and were additionally corrupted by an NLOS bias.

To make a reasonable conclusion about the performance of the algorithm, each scenario was tested using 100000 different positions of the transmitter and the receivers. They were randomly placed on a $[1000 \times 1000]m^2$ plane. For the evaluation, two types of charts are shown, the cumulative distribution function (CDF) of the estimation error, and the root mean square error (RMSE). The RMSE is plotted over varying noise standard deviation or varying Non Line of Sight bias.

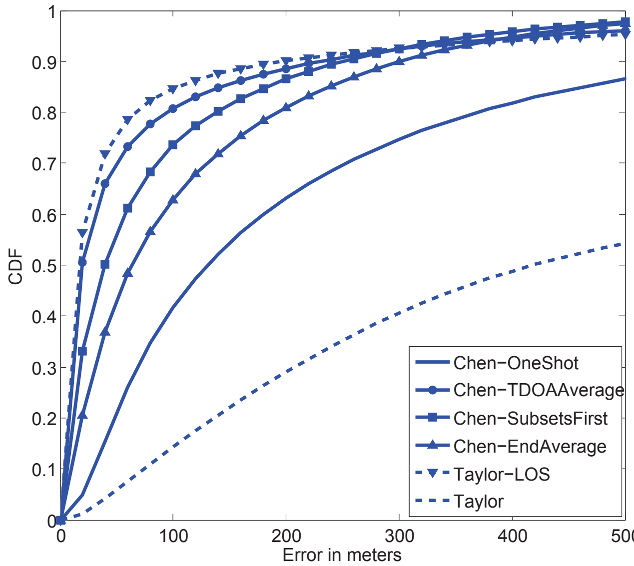

Fig. 3. CDF of position error in meters (noise stdv. = 50m) , 1NLOS receiver with uniformly distributed bias $[0, 1000]m$

Figure 3 shows the results in a scenario with 4 Line of Sight (LOS) receivers and 1 NLOS receiver. The standard deviation of the noise was set to 50m and the NLOS bias was uniformly distributed between $[0, 1000]m$. The figure shows the CDFs of the estimation error for the different averaging techniques that were mentioned before. It also shows the results of the least squares estimation without further residual combining (Taylor) and the results of Chen's weighting algorithm without time averaging (Chen-OneShot). As we can see, averaging the TDOA measurements before calculating the position estimates leads to the best results (Chen-TDOAAverage). Averaging the subset results instead (Chen-SubsetsFirst) also shows good results, but would additionaly cost more computations and is therefore not reasonable. The difference in performance between those two lies in the effects of linearizing the least squares problem in the Taylor Series Estimation to maintain low computational complexity. These linearization effects lead to biased results that can not be totally cancelled out through averaging. On the other hand, we can see that using the residual weighting algorithm needs good subsets estimates to obtain better results. Therefore, the additional averaging is necessary. The performance of the algorithm without further averaging is much worse.

Last but not least, we compare the results to the estimation using only LOS sensors. The algorithm using only LOS receivers (Taylor-LOS) produces the best results, but its performance is close to Chen's method after the measurement averaging.

Figures 4 and 5 show the RMSE over varying standard deviation of the noise and varying NLOS bias. Again, 5 receivers were simulated with one containing an NLOS bias. The results show 2 things: the difference in performance between the least squares estimator (Taylor) and the RWA increase with increasing NLOS error. The larger NLOS errors lead to bigger biases of the least squares estimates, and therefore lead to much larger residuals. Figure 5 also shows that in case of Line of Sight mitigation for all receivers, the least squares estimate has the smallest RMSE because it optimizes the result using all the information. Figure 4 shows the decreasing performance of the RWA with larger standard deviation of the additive noise. This goes back to the assumption Chen made about the residuals of NLOS receivers. With increasing noise, the difference between LOS and NLOS receivers isn't clear in the residuals. Comparing the results in Figure 5 to the estimation using only LOS receivers shows again that with little NLOS bias up to 100 meters, Chen's method performs better when considering random setups.

Figure 6 shows that RWA doesn't perform well when none of the subsets has LOS to the transmitter. At least one of the subsets estimates should yield a bias-free result for the algorithm to work. As we mentioned before, the minimum subset size we need for the two-dimensional case is 4. This means that we need at least 4 receivers with LOS to the transmitter, only then will we have at least one good result among the subset results.

V. Conclusion and Future Work

In this paper, the RWA algorithm was used to analyze different aspects of utilizing consecutive and geometrically distributed TDOA measurements. First, it was shown that additional time averaging is necessary to obtain good results. For that matter, different possibilites of averaging consecutive measurements were tested. The best results along with the lowest computational complexity are achieved through averaging the TDOA measurements.

On the other hand, it was shown that the least squares estimation is sufficient when there's no or little NLOS bias.

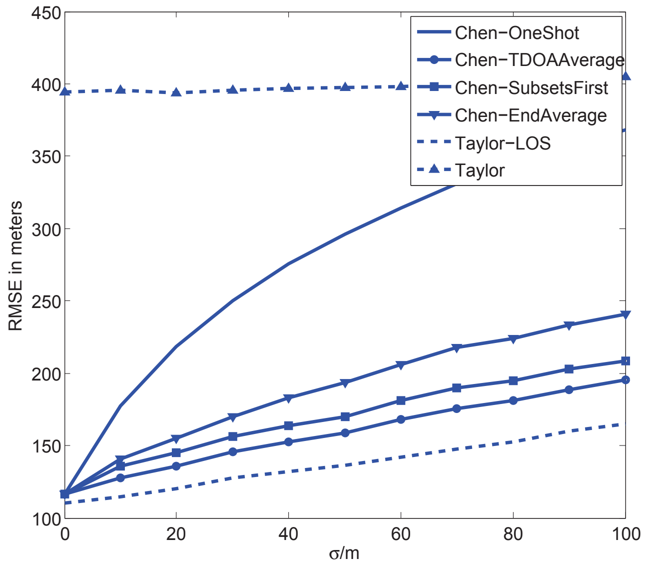

Fig. 4. RMSE over varying noise stdv., 1 NLOS receiver with offset = 500m

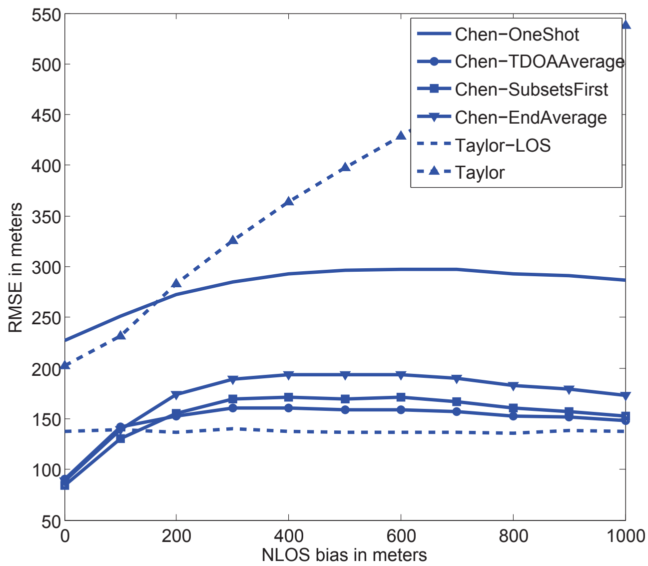

Fig. 5. RMSE over varying NLOS bias, noise stdv. = 50 m

The RWA should be applied when at least 4 receivers have a direct path to the transmitter. If the available receivers are distributed in a way that guarantees at least 4 Line of Sight receivers for each possible transmitter position, then the NLOS problem can be easily solved using the RWA. For the subset sizes, the minimum number was chosen to be 4, while the maximum number should be $N-1$ assuming there is at least one NLOS receiver.

Another important result was the comparison to algorithms that detect and eliminate NLOS receivers. Assuming that these algorithms always detect the right receivers, we have plotted their performance in comparison to the RWA. The results showed a slightly better performance. Nevertheless, if we can

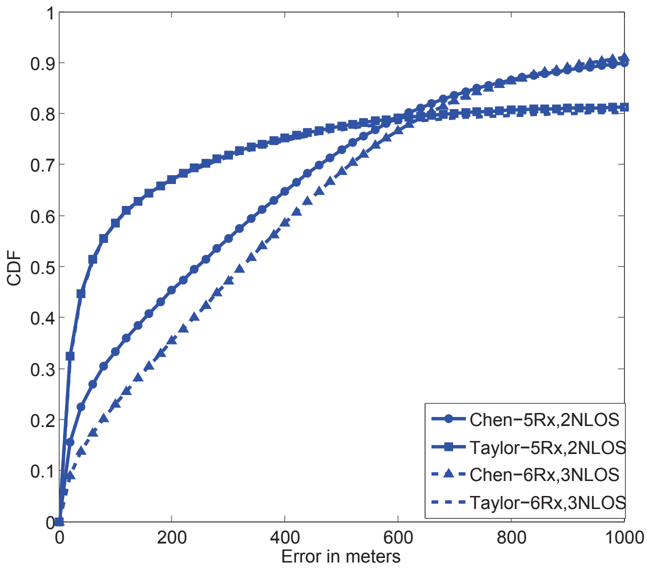

Fig. 6. CDF of RMSE with 5 and 6 receivers, and with 2 and 3 NLOS receivers

not assume that the receiver setup is geometrically convenient, then RWA could be the right choice there, especially if we can assume that the NLOS biases aren't that big.

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