Hopping strategies for Adaptive FH-CDMA Ad Hoc Networks under External Interference

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***Abstract*—We discuss the performance of an adaptive FH-CDMA ad hoc network under the influence of external interference where node positions are modeled by a homogeneous Poisson point process. The optimum channel assignment that balances internal network interference due to spatial reuse and external interference is derived analytically for a path loss and Rayleigh fading model. The performances of the resulting hopping strategies are then compared to various suboptimal hopping strategies such as non-adaptive hopping and min-max allocation with constant QoS. It is found that adaptivity offers a benefit only at low to medium node densities and that a good suboptimal strategy can be based on hard exclusion of bad channels (thresholding) with optional min-max allocation to balance the load of active channels.**

I. Introduction

The study of large dense wireless ad hoc networks is a topic of recent research interest. In such dense wireless networks, the limiting factor to transmission rates is interference, be it self-interference due to spatial channel reuse or external interference from other sources.

Most wireless systems today operate in a power limited and not bandwidth limited regime. Also, various technical difficulties complicate the design of very broadband wireless systems: due to issues such as necessary dynamic range, power consumption or necessary co-site interference attenuation, in practical implementations of wireless systems the total available tuning bandwidth is often much greater than the effective receiver bandwidth of the physical layer. These facts motivate the study of multi-channel medium access techniques in wireless ad hoc networks.

In this paper we consider, within a geometrical model based on a homogeneous Poisson point process (PPP), an adaptive slow FH-CDMA system capable of adapting channel access probabilities (and hence, the hopping pattern) according to the *external* interference experienced in the channels and internal interference generated by other nodes. Due to fast variations in the spatial configuration of transmitting nodes[1] adaption takes into account the *expected* spatial interference. The questions to be answered are the following: What are the highest possible gains of adaptive FH-CDMA and what are the gains of practical suboptimal strategies?

For prior related work addressing the properties of an interference field of a homogeneous PPP spatial node configuration, see [1], [2], [3] and references therein.

[1]Such variations naturally arise when nodes change from transmitting to receiving and vice-versa, or when the network exhibits some mobility.

The remainder of this paper is structured as follows. In Section II the system model is introduced. The transmission capacity of networks under external interference is derived in Section III for a Rayleigh fading and path loss interference model. Section IV compares the considered strategies, while Section V offers concluding remarks.

II. System model

A. Geometry, Channel and Receiver Model

A network consists of nodes distributed in the plane. The total bandwidth B available for communication is split into M orthogonal channels. At each time instance a subset of these nodes transmits in a certain channel $m \in \mathbb{M} \triangleq [1, \ldots, M]$. Assuming that slotted Aloha is employed and channel access is uncoordinated among the nodes, we model the transmitter positions by a homogeneous independently marked PPP $\Phi \triangleq \{(X_i, m_i)\}_{i=1}^{\infty}$ of intensity λ, where the $X_i \in \mathbb{R}^2$ denote the locations of transmitters and the m_i are the marks attached to the X_i that indicate the associated channel. We assume that by the time of transmission, each transmitter randomly chooses a channel according to the *channel access probabilities* $\mathbf{p}$ for transmitting its message. For convenience, we define $\Phi_m := \{(X_i, m_i) | X_i \in \Phi, m_i = m\}$ as the point process counting only those transmitters which transmit in channel m. The intensity of Φ_m is denoted by $\lambda_m = p_m \lambda$.

Each transmitter has an associated receiver at a distance[2] r and transmits with power ρ. The transmitted signals are attenuated by path loss and may also be subject to Rayleigh fading. The path loss between two points x, y is given by $\|x - y\|^{-\alpha}$, with $\alpha > 2$. We assume that any interference is treated as white noise.

Due to the homogeneity of the PPP, the resulting interference field obeys the same statistics at any point in the plane. This allows characterizing the performance of the whole network by the performance of a *typical* transmission. Therefore, we place a probe receiver in the origin and an associated probe transmitter r units away at location x. The instantaneous SINR at the probe receiver in channel m is given

[2]This assumption poses a restriction on the system model but allows for analytical tractability. The effect of r being different for every transmitter-receiver pair on the results is investigated with numerical simulations (see Fig. 3).

by

$$\begin{aligned}\mathrm{SINR}_m &\triangleq \frac{\rho\, G_0 r^{-\alpha}}{G_N\, N_m + \sum_{\Phi_m\setminus\{x\}} \rho\, G_i \|X_i\|^{-\alpha}} \\ &= \frac{G_0}{G_N\, \mathrm{NSR}_m + r^{\alpha} \sum_{\Phi_m\setminus\{x\}} G_i \|X_i\|^{-\alpha}}, \end{aligned} \quad (1)$$

where N_m is the noise level in channel m due to the external interference, $\mathrm{NSR}_m \triangleq \frac{N_m}{\rho r^{-\alpha}}$ is the mean noise-to-signal ratio in channel m. The variables G_0, G_N and G_i are exponentially distributed with unit mean and capture the random fluctuations in the received power due to Rayleigh fading at the probe receiver. By setting $G_N \equiv G_0 \equiv G_i \equiv 1$, (1) reduces to the path loss model.

The outage probability of the probe link operating in channel m is given by the reduced Palm probability [4]

$$\begin{aligned} q_m(\lambda_m) &\triangleq \mathbb{P}^{!x}\left[\mathrm{SINR}_m < \beta\right] \\ &\overset{(a)}{=} \mathbb{P}\left[\mathrm{SINR}_m < \beta\right], \end{aligned} \quad (2)$$

where β is the required SINR threshold and $\mathbb{P}^{!x}$ is the probability measure with respect to the point process $\Phi_m \cup \{x\}$ without counting the point x, as the probe transmitter does not contribute to the interference seen by the probe receiver. (a) follows from Slivnyak's Theorem, which states that $\mathbb{P}^{!x}[\cdot] = \mathbb{P}[\Phi \in \cdot]$, if Φ is a PPP [4]. We define the overall outage probability as

$$q(\lambda, \mathbf{p}) \triangleq \sum_{m=1}^{M} p_m q_m(p_m \lambda), \quad (3)$$

i.e., we consider the *average* outage probability associated with the channel access probabilities $\mathbf{p}$. This expression can be interpreted as the *effective* outage probability of a single transmission.

Our primary metric of interest is the transmission capacity (TC) [3], which is the density of concurrent transmissions weighted by the success probability associated with this density, i.e.,

$$c(\lambda, \mathbf{p}) \triangleq \lambda(1 - q(\lambda, \mathbf{p})). \quad (4)$$

It is easy to see that for two $\mathbf{p}_1 \neq \mathbf{p}_2$, $c(\lambda, \mathbf{p}_1) \neq c(\lambda, \mathbf{p}_2)$ in general, since $q(\lambda, \mathbf{p}_1) \neq q(\lambda, \mathbf{p}_2)$. Therefore, the value of $c(\lambda, \mathbf{p})$ strongly depends on how the channel access probabilities $\mathbf{p}$ are chosen.

B. Internal and external interference: Optimizing channel access

Internal interference in a channel is the aggregated interference generated by other nodes of the same network transmitting in the same channel. External interference is interference generated by sources outside the network. Both interference sources affect the outage probability of a transmission in a certain channel.

Our degree of freedom is the channel access probability distribution $\mathbf{p}$. If we choose to increase the load on one channel by assigning more probability mass to it, we will increase the outage probability in that channel. We are now looking for the optimum distribution $\mathbf{p}_{\text{opt}}$ that maximizes the TC and hence balances internal and external interference.

Definition 1. *We define by $\mathbf{p}_{opt}$ the channel access probabilities for which the average outage probability $q(\lambda)$ is minimal given some λ.*

Lemma 1. *The average outage probability $q(\lambda)$ is a strictly monotonically increasing function of λ if $\mathbf{p} = \mathbf{p}_{opt}$ at every λ.*

Proof: Proof by contradiction: Suppose that $q(\lambda_1, \mathbf{p}_1) \geq q(\lambda_2, \mathbf{p}_2)$ for some arbitrary $\lambda_1 < \lambda_2$. From the definition of $\mathbf{p}_{\text{opt}}$ it follows that both $q(\lambda_1, \mathbf{p}_1)$ and $q(\lambda_2, \mathbf{p}_2)$ are minimal at λ_1 and λ_2, respectively. Analyzing $\frac{\partial q_m(\lambda p_m)}{\partial \lambda}|_{\lambda=\lambda_2}$ we see that this is, due to the nature of q_m, always positive for fixed $\mathbf{p} = \mathbf{p}_2$. Thus, going "backwards" from λ_2 to a point λ, we have $q(\lambda, \mathbf{p}_2) < q(\lambda_2, \mathbf{p}_2)$. After reaching the point $\lambda = \lambda_1$, we observe that $q(\lambda_1, \mathbf{p}_2) < q(\lambda_2, \mathbf{p}_2)$. Using the initial assumption, we finally have

$$q(\lambda_1, \mathbf{p}_2) < q(\lambda_2, \mathbf{p}_2) < q(\lambda_1, \mathbf{p}_1),$$

which is a contradiction to the assumption that $\mathbf{p}_1$ is optimal and hence, $q(\lambda_1, \mathbf{p}_1)$ is minimal. Therefore, $q(\lambda_1, \mathbf{p}_1) < q(\lambda_2, \mathbf{p}_2)$ always holds. ■

Lemma 2. *The maximization of $\lambda(q)$ for a given q is equivalent to minimizing $q(\lambda)$ for a given λ.*

Proof: From Lemma 1 we know that the average outage probability $q(\lambda)$ with $\mathbf{p} = \mathbf{p}_{\text{opt}}$ is a strictly monotonically increasing function of λ. Hence, for any pair (λ', q') generated by $\mathbf{p}_{\text{opt}}$, we have that $\lambda' = \max_{\mathbf{p}}\{\lambda(q', \mathbf{p})\}$ and $q' = \min_{\mathbf{p}}\{q(\lambda', \mathbf{p})\}$ and the Lemma follows. ■

Using Lemma 2 we can now transform the problem of maximizing the TC into the problem of minimizing the average outage probability $q(\lambda, \mathbf{p})$ over $\mathbf{p}$ given some λ.

C. Optimization Problem 1

The first problem strives to minimize the average outage probability $q(\lambda, \mathbf{p})$ from (3) as follows:

$$\mathbf{p}_{\text{opt}} = \arg\min_{\mathbf{p}} \sum_{m=1}^{M} p_m q_m(p_m \lambda) \text{ s.t. } \|\mathbf{p}\|_1 = 1,\, p_m \geq 0\,. \quad \text{(P1)}$$

By Lemma 2, the solution of (P1) yields the transmission capacity of (4). If all q_m are convex, the problem can be solved with convex optimization [5, pp. 20ff]. If the q_m are non-convex, the optimization problem can generally only be solved heuristically. Due to their nature as cumulative density functions, the q_m are monotonically increasing in λ_m, however not necessarily convex.

D. Optimization Problem 2

Practical systems will strive to have the same expected packet error probability, every time channel is accessed, hence assuring constant quality of service (QoS). An optimization approach that achieves this is based on minimizing the maximum weighted outage probability associated with the M channels:

$$\mathbf{p}_{\text{opt}} = \arg\min_{\mathbf{p}} \max_{m} p_m q_m(p_m \lambda) \text{ s.t. } \|\mathbf{p}\|_1 = 1,\, p_m \geq 0\,. \quad \text{(P2)}$$

From [5, Theorem 2.4.1], we know that for the global minimum of (P2), $\forall i, j : \lambda_i q_i(\lambda_i) = \lambda_j q_j(\lambda_j) = \text{const}$ holds: The access probability weighted packet error probability in every channel remains constant. The solution of (P2) hence yields the *transmission capacity under constant QoS*. Due to the monotonicity of $\lambda_m q_m(\lambda_m)$, this optimization problem has a unique global minimum and efficient algorithms exist to solve it numerically [5, pp. 31ff].

III. Transmission capacity under external interference

In the following, we consider well known interference functions q_m arising in a path loss only interference field and in a Rayleigh block fading interference field. For the pure path loss model with $\alpha = 4$, q_m is given by [3]

$$q_m^{\text{pl}}(\lambda_m) \triangleq 2Q\left(\lambda_m \xi_m^{\text{pl}}\right) - 1, \tag{5}$$

where $\xi_m^{\text{pl}} \triangleq \sqrt{\frac{\pi}{2}} \frac{\pi r^2}{\sqrt{\gamma_m}}$ and

$$\gamma_m \triangleq \begin{cases} \frac{1}{\beta} - \text{NSR}_m & , \frac{1}{\beta} - \text{NSR}_m > 0 \\ 0 & , \text{otherwise.} \end{cases} \tag{6}$$

Similarly, for the Rayleigh block fading model, q_m is given by [6]

$$q_m^{\text{rl}}(\lambda_m) \triangleq 1 - e^{-\lambda_m \Delta} \xi_m^{\text{rl}}, \tag{7}$$

where $\Delta \triangleq 2\pi^2 \frac{r^2 \beta^{2/\alpha}}{\alpha \sin 2\pi/\alpha}$ and

$$\xi_m^{\text{rl}} \triangleq \begin{cases} \frac{1}{1+\beta \text{NSR}_m} & , g_N \sim \text{Exp}(1) \\ e^{-\beta \text{NSR}_m} & , g_N \equiv 1. \end{cases} \tag{8}$$

A. Optimal strategy for path loss only model $q_m = q_m^{pl}$

The adaptive channel allocation minimizes the outage probability by adapting p_m and hence λ_m according to the quality level γ_m of the respective channels. The problem (P1) can be written as

$$\min_{\mathbf{p}} \sum_{m=1}^{M} p_m q_m(p_m \lambda) = \min_{p_o, p_{L+1}, \ldots, p_M} p_o + \sum_{m=L+1}^{M} p_m q_m(p_m \lambda) \tag{9}$$

$$\text{s.t. } p_o + \sum_{m=L+1}^{M} p_m = 1, \quad p_o, p_m \geq 0 \,, m = L+1, \ldots, M,$$

where $1, \ldots, L$ are the indices[3] of channels for which $\gamma_m = 0$ and hence $q(\lambda_m) = 1$, and p_o is the channel access probability assigned to these channels. The probability p_o can be interpreted as the optimum back-off probability needed to maximize transmission capacity by reducing internal interference.

The optimizing problem (9) is generally non-linear and non-convex with non-linear non-convex monotonically increasing objective function and linear equality and inequality constraints. If the function q can be expressed analytically, as in the two cases considered here, the objective function of (P1) can be tested for convexity by showing positive semi-definiteness of the Hessian matrix. Additional constraints for small λ_m, and hence total λ, can then assure convexity. In the following, we will derive the optimum solution under these additional constrains, focusing on small λ and thus small (practically relevant) outage probabilities. We note three relevant observations:

Lemma 3. *The optimization problem* (P1) *with $q_m = q_m^{pl}$ and $\alpha = 4$ is convex, if $\forall m : 0 \leq \lambda_m \leq \frac{\sqrt{2}}{\xi_m^{pl}}$.*

A proof is given in the appendix.

Lemma 4. *The optimization over p_o can be performed separately after finding the solution for $p_{L+1}, \ldots, p_M$.*

Proof: Optimization over $p_{L+1}, \ldots, p_M$ of the objective function of (9) results in a function that depends on p_o. The minimum of this function is also the global minimum. ■

Lemma 5. *A necessary condition for an extremum of (P1) with $q_m = q_m^{pl}$ is that $\forall i : q_m(\lambda_i, \gamma_i) = \tau$, $\tau \in [0, 1]$.*

A proof is given in the appendix.

Since $q(\lambda_m)$ is strictly monotonically increasing with respect to λ_m, for a given p_o minimizing $q(\lambda_m)$ is equivalent to minimizing its argument. With this fact and Lemma 4 and Lemma 5, the optimization problem over $p_{L+1}, \ldots, p_M$ for a given p_o can hence be written as

$$\mathbf{p}_{\text{opt}}(p_o) = \arg \min_{p_{L+1}, \ldots, p_M} \tau \tag{10}$$

$$\text{s.t. } \tau - \frac{\gamma_m}{(p_m \lambda)^{\frac{\alpha}{2}}} = 0,$$

$$p_o + \sum_{m=L+1}^{M} p_m = 1, \quad p_o, p_m \geq 0 \,, m = L+1, \ldots, M.$$

The problem in (10) is a convex optimization problem with linear objective function and convex equality constraints and linear inequality constraints. This optimization problem can be analytically solved with the help of Karush-Kuhn-Tucker (KKT) conditions[4]. Using Lemma 3, we have the following result:

Theorem 1 (TC with optimal strategy, path loss only)**.** *The solution of the optimization problem* (P1) *with $q_m = q_m^{pl}$ in the convex region is given by*

$$p_m^*(\lambda_o) = \frac{\gamma_m^{\frac{2}{\alpha}}}{\sum_{m=L+1}^{M} \gamma_m^{\frac{2}{\alpha}}} (1 - p_o) \text{ for } m = L+1, \ldots, M. \tag{11}$$

and $p_m^* = \frac{p_o}{M-L}$ otherwise.

Proof: The solution follows by solving the standard KKT equations. ■

Note that (11) holds for all α, it is necessarily a local optimum. It will be a global optimum for all values of α if conditions on the per channel density, similar to those of Lemma 3, are met (in case of $\alpha = 4$ convexity is assured by Lemma 3). To determine p_o, we numerically solve (9) with the obtained $p_m^*(p_o)$.

[3] A reordering might be necessary.

[4] See e.g. [5, Section 2.1] for an introduction.

B. Optimal strategy for Rayleigh fading model $q = q^{rl}$

Analogously to the path loss case, we state the following observation:

Lemma 6. *The optimization problem* (P1) *with* $q_m = q_m^{rl}$ *is convex, if* $\forall m : \lambda_m \Delta \leq 2, m = 1, \ldots, M$.

A proof is given in the appendix.

Theorem 2 (TC with optimal strategy, Rayleigh fading)**.** *The optimal* $p_m^* \in \mathbf{p}_{opt}$ *of the optimization problem* (P1) *with* $q_m = q_m^{rl}$ *in the region* $\lambda_m \Delta \leq 2$ *are given by*

$$p_m^* = \max\left[0, \frac{1}{\lambda\Delta}\left(1 - \mathcal{W}\left(\frac{\nu^* e}{\lambda \xi_m^{rl}}\right)\right)\right], \quad m = 1, \ldots, M, \tag{12}$$

where ν^* *is the solution of*

$$\sum_{m=1}^{M} \max\left[0, \frac{1}{\lambda\Delta}\left(1 - \mathcal{W}\left(\frac{\nu e}{\lambda \xi_m^{rl}}\right)\right)\right] - 1 \stackrel{!}{=} 0. \tag{13}$$

and $\mathcal{W}(\cdot)$ *denotes the principal branch of the Lambert-W function.*

Proof: The solution follows by solving the standard KKT equations. ■

IV. Numerical evaluation

A. Suboptimal channel access strategies

In the following, we describe the following suboptimal channel access strategies: Naïve, best channel, threshold-based and min-max optimized threshold-based channel access.

1) Naïve channel access: For the naïve strategy, every node selects a channel with equal probability, i.e., $\mathbf{p} = [\frac{1}{M}, \ldots, \frac{1}{M}]^T$. The resulting density in each channel is $\lambda_m = \frac{\lambda}{M}$ for all m. This corresponds to standard FH-CDMA.

2) Best channel access: For the best (single) channel access strategy, only the best available channel, i.e., the channel with the least NSR_m, is used. The corresponding intensity λ_m is given by

$$\lambda_m = \begin{cases} \lambda & , m = \arg\min_n \{\text{NSR}_n | n = 1, \ldots, M\} \\ 0 & , \text{otherwise.} \end{cases} \tag{14}$$

3) Threshold-based channel access: With the threshold based channel access strategy, the best K channels as well as all remaining channels with sufficient quality are used. The criterion for a channel m to be active is $\text{NSR}_m \leq \kappa$, where κ denotes the quality threshold. Let the set of active channels be denoted by

$$\mathcal{K} \triangleq \{m \in \mathbb{M} : \text{NSR}_m \leq \kappa \vee \text{NSR}_m \text{ among the } K \text{ smallest}\}$$

and let $|\mathcal{K}|$ with $|\mathcal{K}| \geq K$ be the number of active channels, then this channel access strategy assigns transmission density to the channels according to

$$\lambda_m = \begin{cases} \frac{\lambda}{|\mathcal{K}|} & , m \in \mathcal{K} \\ 0 & , \text{otherwise.} \end{cases} \tag{15}$$

This thresholding strategy is comparable to the mechanism implemented in IEEE 802.15.1 [7].

4) Min-max optimized threshold-based channel access: In this optimized version of threshold-based channel access, the channel access probabilities for the active channels are determined by solving the min-max problem over all *active* channels, i.e.,

$$\begin{aligned} \min_{\mathbf{p}} \max_{m} \; & p_m q_m(p_m \lambda) \\ \text{s.t. } & p_m \geq 0 \text{ if } m \in \mathcal{K}, \\ & p_m = 0 \text{ if } m \in \mathcal{K}^c, \\ & \|\mathbf{p}\|_1 = 1. \end{aligned}$$

Again, min-max channel allocation assures constant QoS.

B. Comparison of channel access strategies

Parameter	r	β	α	M	$\overline{\text{NSR}}$	κ	K
Value	10	1	4	10	-5 dB	5 dB	3

TABLE I
EVALUATION PARAMETERS

For numerical evaluation and comparison of the strategies, the parameters given in Table I with NSR $\sim$ Exp and mean $\overline{\text{NSR}} = -5$ dB were used[5]. In the non-convex region, that is for high λ and q, the $\mathbf{p}_{\text{opt}}$ were obtained numerically using global optimization heuristics.

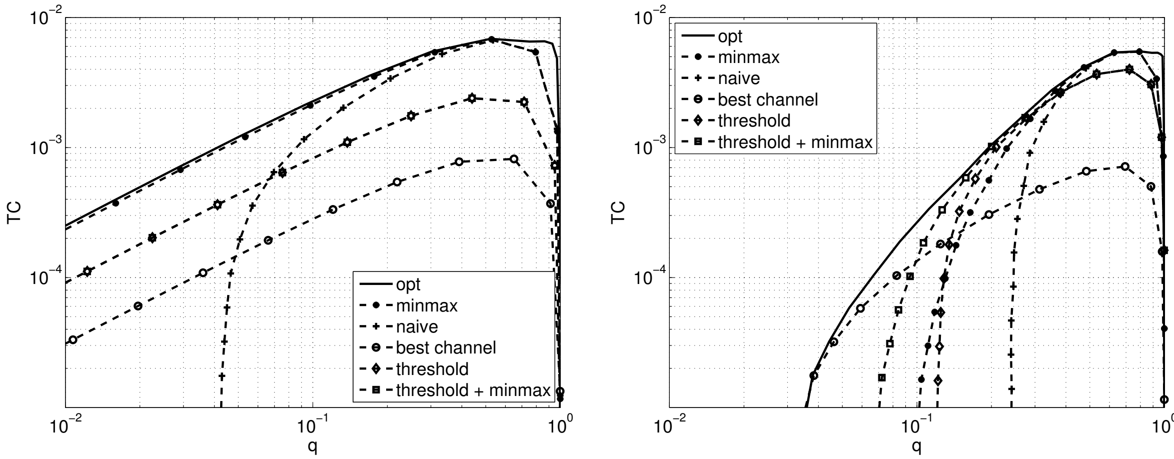
Fig. 1(a) and Fig. 1(b) show the TC for the path loss and Rayleigh fading model without fading between receiver and external interference ($G_N \equiv 0$). Comparing the two figures, we observe that the TC for Rayleigh fading is generally lower than for the path loss model which is consistent with findings in the single channel case [3].

Furthermore we observe that the channel access strategies behave qualitatively differently in the Rayleigh fading model as compared to the path loss model: all strategies exhibit a non-vanishing outage probability even for $\lambda \to 0$. This is due to the fact that outage still may occur, even for very low densities, due to bad fades.

At low outage probabilities, and hence low λ, the optimal strategy is to choose the best channel since this minimizes overall outage probability. For the path loss model, this effect is not observed: outage probability is unaffected by the quality of good channels. Here, equally balancing internal and external interference is optimal even when λ becomes arbitrarily small. As a result, the optimal and the min-max strategy yield the same solution at small q, while thresholding (with and without min-max optimization) performs slightly worse because not all "alive" channels are used. It should be noted that the transmission capacity metric does not reveal the performance limit of the latter strategies in the path loss model as it assumes a fixed β. Adaptive modulation and coding could be employed by the transmitters in order to benefit from good channel states.

In the intermediate outage probability region, where internal and external interference are of the same order, we can see that

[5]Note that the assumption NSR $\sim$ Exp has been made ad hoc. However, the exact external interference statistics do not affect the relative performances of the strategies.



(a) Path loss model (b) Rayleigh fading model

Fig. 1. Transmission capacity for various strategies

both the naïve and the best channel strategy do not perform well. Here, thresholding, and in particular thresholding with min-max optimization, yield large performance gains over a wide range and show near-optimality. For Rayleigh fading, its performance is even better than the min-max solution.

In the high outage probability (and high node density) region, the naïve strategy quickly approaches the optimal TC before falling off.

The TC for Rayleigh fading with $G_N \sim \text{Exp}(1)$ is not shown, but the qualitative characteristics of the channel access strategies do not differ from those of Fig. 1(b). The TC with $G_N \sim \text{Exp}(1)$ is increased by approx. 8.5%: If fading between receiver and external interferer is present, the outage probability associated with the external interference is lower than without fading. This can directly be seen by applying the Jensen inequality, $\mathbb{P}[\frac{g_1}{g_2} < a] = 1 - \mathbb{E}[e^{-g_2 a}] \leq 1 - e^{-\mathbb{E}[g_2]a} = 1 - e^{-a}$. Thus, fading between receiver and external interferer is beneficial. Knowledge about the type of this channel can hence be exploited for the calculation of the optimal channel access probabilities in order to increase TC.

Fig. 2(a) and 2(b) show the average outage probability $q(\lambda)$ as well as the average standard deviation of the weighted channel outage probabilities, i.e., $\sigma = \text{std}(p_1 q_1(\lambda_1), \ldots, p_M q_M(\lambda_M))$. For the threshold strategy, only the active channels are considered. It can be observed that σ is nearly constant over a wide range. At (undesirable) high outage probabilities, σ increases fast for the optimal strategy (the bend indicates the end of the convex region) and decreases for the threshold and naïve strategy. At low λ, σ is of the same order than q, which may have a negative effect on QoS guarantees. Note that for both the min-max and the threshold-based min-max strategy $\sigma = 0$.

Finally, Fig. 3 shows the effect of varying transmission distances r while optimizing the channel access probabilities based on a target transmission distance $r_{\text{opt}} = 10$. In both cases of channel models, outage probability increases only marginally and still remains lower than in the case of the suboptimal solutions.

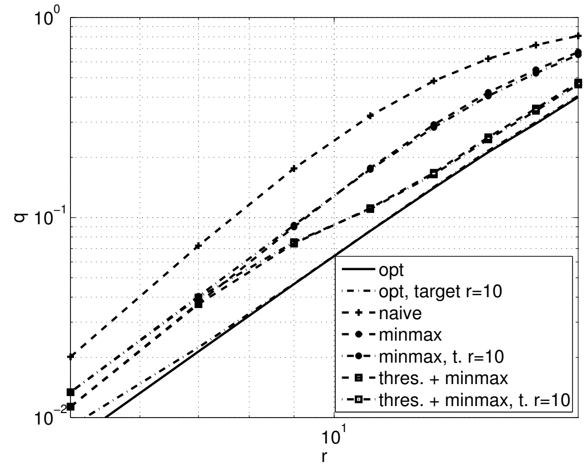


Fig. 3. Influence of varying transmission distance for a fixed optimization target $r_{\text{opt}} = 10$.

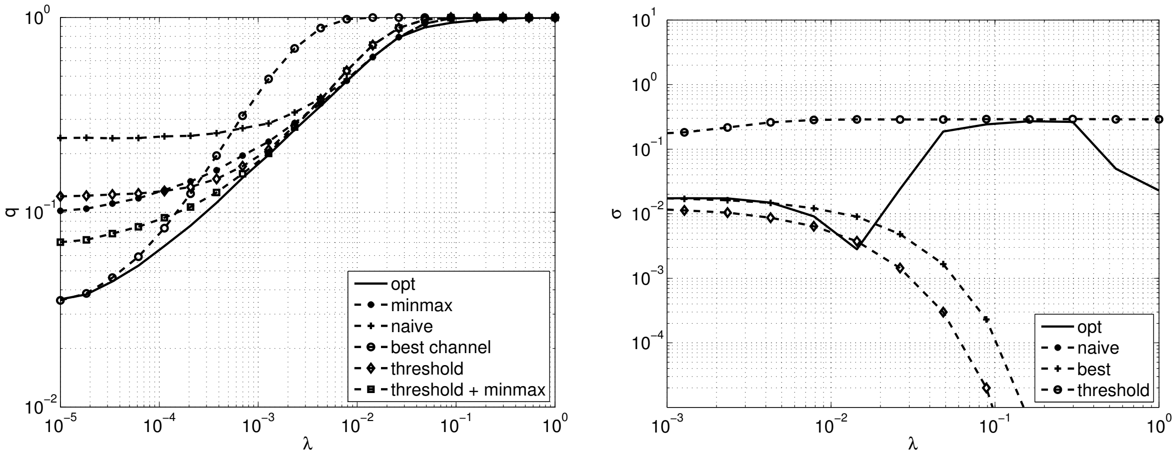
C. Implications for protocol design

For protocol design, two conclusions can be drawn from the analysis and comparison of strategies: First, adaptivity does not result in a gain if the node density is high and hence internal interference dominates. In such high density networks, applying a naïve strategy will then yield a close to optimal result.

Second, if the node density is lower, adaptivity can indeed help: In both the path loss and the Rayleigh model, a min-max strategy, with optional thresholding to exclude bad channels, is a good strategy with the benefit of constant QoS. This is consistent with the interference avoidance mechanisms currently implemented in IEEE 802.15.1.

V. Conclusion

We derived, for the convex low to medium outage region, analytical TC expressions of the given FH-CDMA system for both a Rayleigh fading and path loss model. Suboptimal strategies were compared to those bounds in numerical simulations.



(a) Outage probability

(b) Standard deviation of outage probability

Fig. 2. Outage probability and standard deviation as QoS for various strategies (Rayleigh model without fading of external interference)

Future work will focus on finding analytical expressions and bounds for those suboptimal allocation strategies, in order to create an analytical framework for FH-CDMA ad hoc networks under external interference.

APPENDIX

A. Proof of Lemma 3

We examine the convexity of $c^{\mathrm{pl}} = \sum \lambda_m q^{\mathrm{pl}}(\lambda_m)$. Taking the second partial derivative with respect to λ_m, we find

$$\frac{\partial^2 c^{\mathrm{pl}}}{\partial \lambda_m^2} = \begin{cases} \frac{2}{\pi}\xi_m^{\mathrm{pl}} e^{-\frac{1}{2}(\xi_m^{\mathrm{pl}}\lambda_m)^2}(2-\lambda_m^2(\xi_m^{\mathrm{pl}})^2) & ,i=j, \\ 0 & ,i \neq j. \end{cases} \tag{16}$$

The Hessian matrix $\mathbf{H}$ of the objective function thus has positive elements only for $i = j$, $i \geq L+1$, and is zeros elsewhere. For $\mathbf{H}$ to be positive semi-definite and hence q_m to be convex, $0 < \lambda_m \leq \frac{\sqrt{2}}{\xi_m^{\mathrm{pl}}}$ must hold.

B. Proof of Lemma 5

Let $f_i(\lambda_i) := \frac{\gamma_i}{(\pi r^2 \lambda_i)^{\frac{\alpha}{2}}}$ and $f_j(\lambda_j) := \frac{\gamma_j}{(\pi r^2 \lambda_j)^{\frac{\alpha}{2}}}$. Assume we have an extremum of the objective function of (P1) in the convex region with associated solution $\lambda, \ldots, \lambda_M$. Consider two λ_i and λ_j of the solution for which $\gamma_i \neq 0, \gamma_j \neq 0$. We can write

$$k = \sum \lambda_m q^{\mathrm{pl}}(\lambda_m, \gamma_m) = \lambda_i q^{\mathrm{pl}}(f_i(\lambda_i)) + \lambda_j q^{\mathrm{pl}}(f_j(\lambda_j)) + \mathcal{R}, \tag{17}$$

where $\mathcal{R}$ are all terms independent of λ_i, λ_j. At the solution, the constraint $\sum \lambda_m = \lambda$ has to be fulfilled, so we can set $\lambda_i + \lambda_j = \lambda'$ and write (17) in terms of λ_i. Furthermore, we know that at an extremum the partial derivative with respect to λ_i has to be zero:

$$\begin{aligned} \frac{\partial k}{\partial \lambda_i} &= q^{\mathrm{pl}}(f_i(\lambda_i)) + \frac{\alpha}{2}(q^{\mathrm{pl}})'(f_j(\lambda'-\lambda_i))f_j(\lambda'-\lambda_i) \\ &\quad - q^{\mathrm{pl}}(f_j(\lambda'-\lambda_i)) - \frac{\alpha}{2}(q^{\mathrm{pl}})'(f_i(\lambda_i))f_i(\lambda_i) = 0 \end{aligned} \tag{18}$$

$f_i(\lambda_i) = f_j(\lambda' - \lambda_j)$ is a solution to this equation; we assumed convexity (and strict convexity for $\lambda_{L+1}, \ldots, \lambda_M$) of the objective function, hence it is the only solution. Since this is the case for arbitrary i and j, all $f_m(\lambda_m)$ are equal and the Lemma follows.

C. Proof of Lemma 6

We examine the convexity of $c^{\mathrm{rl}} = \sum \lambda_m q^{\mathrm{rl}}(\lambda_m)$. The elements of the Hessian matrix $\mathbf{H}$ are given by

$$\frac{\partial^2 c^{\mathrm{rl}}}{\partial \lambda_i \, \partial \lambda_j} = \begin{cases} \lambda^2 \Delta \xi_i^{\mathrm{rl}} e^{-\lambda_i \Delta}(2-\lambda_i\Delta) & ,i=j \\ 0 & ,i \neq j. \end{cases} \tag{19}$$

The matrix $\mathbf{H}$ is positive semi-definite if all minors are non-negative. From (19) it can be observed that $\mathbf{H}$ is a diagonal matrix. Hence,

$$\prod_{m=1}^{k} \lambda^2 \Delta \xi_m^{\mathrm{rl}} e^{-\lambda_m \Delta}(2-\lambda_m\Delta) \geq 0 \tag{20}$$

must hold for all $k = 1, \ldots, M$. Since all factors are non-negative, (20) is true only if $2 - \lambda_m \Delta$ is non-negative for all $m = 1, \ldots, M$.

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