A Weighted Least Squares Algorithm for Passive Localization in Multipath Scenarios

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***Abstract*—This paper presents a weighted least squares algorithm for passive localization using Time Difference of Arrival in multipath scenarios. The algorithm is based on the direct or one-step approach for position estimation using distributed sensors and requires no knowledge about the number of received multipaths, the transmitted signal or the transmit time. Delayed signal paths arriving due to multipath propagation are considered and treated as interference to the localization problem. Simulation results show a better and more robust performance of the algorithm, compared to conventional two-step localization algorithms.**

I. Introduction

Passive localization describes the problem of estimating the position of a signal source without knowledge of the transmitted signal or the transmit time. This can be employed for example by frequency regulators aiming at finding unlicensed transmitters or for a variety of security and emergency scenarios. Depending on the available hardware, the information about the transmitter position can be estimated passively by measuring the time difference of arrival, the angle of arrival or the received signal strength difference.

Time difference of arrival (TDoA) offers a good compromise between low cost hardware and reliable estimates. In [1], a low cost TDoA system was presented. Based on GPS-synchronized software defined radios, the presented setup is able to deliver synchronized IQ-data received by distributed sensors with known positions.

The challenges facing TDoA systems can be categorized in: (i) additive white Gaussian noise (AWGN), (ii) multipath propagation, (iii) non-line-of-sight (NLOS) propagation. While it is true that NLOS is part of the multipath scenario, nevertheless, the two problems are modeled and analyzed separately in TDoA systems. For the simple case of AWGN, the estimation problem can be solved by calculating the cross-correlation between pairs of received signals and obtaining the TDoA measurement by detecting the correlation peak [2]. The estimated TDoAs are fed to a positioning algorithm, for example least squares algorithms presented in [3] and [4]. For the AWGN scenario, this two-step method results in accurate and reliable estimates.

A sensor receiving a multipath propagated signal observes multiple delayed versions of the signal in its observation window. A simple cross-correlation of the observed signal would result in multiple correlation peaks. The difficulty in estimating the first arrival path from this cross-correlation lies in the fact that the strongest peak doesn't necessarily represent the direct path. On the other hand, peaks can overlap in an unresolvable way where the individual peak is no more distinct. The obtained TDoA estimates tend to be biased in these scenarios. In the literature, different methods aiming at estimating the time delays resulting from multipath propagation are based on maximizing the likelihood function of the presented problem as in [5] and [6] or by using super-resolution methods as in [7]. These algorithms assume a known number of received paths and perform well whenever the multipath components are well separated in time. This assumption holds whenever the multipath delays are large compared to the signal correlation peak, which depends on the signal bandwidth as well as the channel. A narrowband signal propagating through an urban channel will not result in resolvable correlation peaks.

Assuming that the estimated TDoAs are biased due to the absence of a strong first arrival path of the signal, [8] and [9] presented algorithms that aim at identifying and mitigating the NLOS error by weighting the estimated TDoAs according to their reliability or by eliminating the identified NLOS TDoA estimates. These algorithms perform well whenever there are enough line-of-sight (LOS) TDoA estimates. As a consequence, there is still a need for passive localization methods using TDoA in multipath scenarios.

The mentioned methods are all based on the two-step estimation procedure, the first step estimating the TDoAs from the received signals and the second step estimating the position from the obtained TDoAs. Alternatively, so called one-step methods have also been presented as a good and reliable approach to the problem. In the one-step methods, a position is estimated directly from the received signals. The algorithms are based on a grid search, which is the reason why, in good conditions, the two-step methods are preferred. In [10], Weiss presented a direct positioning method for narrowband transmitters. The results showed a better performance of his method than usual one-step methods, especially at low SNR. In [11], the one-step maximum likelihood estimator for passive localization was presented for the case of known and unknown transmitted signal. In this paper we exploit the advantage of the one-step least squares solution for the complex scenario of passive localization in multipath channels and present a novel algorithm based on it. We show how, by preprocessing the received multipath signals, we can achieve better positioning results than conventional two-step approaches.

The paper is organized as follows. Section II introduces the system model as well as the least squares solution. Section III describes the developed algorithm. Section IV shows and analyzes simulation results. Section V concludes the paper.

II. System Model

The passive localization system consists of M distributed sensors with positions $\mathbf{x}_i = [x_i, y_i]^T \quad i = 1, 2, ...M$ and an unknown transmitter at coordinates $\mathbf{x} = [x_T, y_T]^T$. First, we will present the AWGN scenario and its least squares solution. The received and sampled signal at sensor i can be modeled as:

$$r_i(n) = \alpha_i s(n - t_0 - \tau_i) + \eta_i(n), \quad n = 0, 1, ..., K-1 \quad (1)$$

whereas $s(n)$ is the unknown transmitted signal, t_0 is the transmit time, τ_i is the propagation delay and η_i is a white Gaussian random noise at sensor i. The propagation delays are functions of the emitter position given as:

$$\tau_i(\mathbf{x}) = t_0 + \frac{\sqrt{(x_T - x_i)^2 + (y_T - y_i)^2}}{c} \quad (2)$$

with c being the propagation speed. To be able to separate the signal from the parameters that need to be estimated, the sampled signal is transformed to the frequency domain and the least squares solution is given by minimizing the following cost function [10]:

$$Q(\mathbf{x}) = \sum_{i=1}^{M} \sum_{k=0}^{K-1} |R_i(k) - \alpha_i S(k) e^{(t_0 + \tau_i(\mathbf{x})) w_k}|^2 \quad (3)$$

whereas $R_i(k)$ and $S(k)$ are the Fourier transforms of $r_i(n)$ and $s(n)$ and $w_k = \frac{-j2\pi k}{K}$. The passive localization scenario assumes unknown transmitted signal and transmit time, resulting in ambiguity of this solution as it was shown in [11]. Using one of the received signals as reference signal resolves the ambiguity. Without loss of generality, we define signal $r_1(n)$ as our reference signal and rewrite the least squares solution as follows:

$$\bar{Q}(\mathbf{x}) = \sum_{i=2}^{M} \sum_{k=0}^{K-1} |R_i(k) - \beta_i R_1(k) e^{\Delta\tau_i(\mathbf{x}) w_k}|^2 \quad (4)$$

whereas: $\beta_i = \frac{\alpha_i}{\alpha_1}$ and $\Delta\tau_i = \tau_i - \tau_1$. Minimizing the least squares equation yields for β_i:

$$\beta_i = (\mathbf{\Phi}_i(\mathbf{x})^H \cdot \mathbf{\Phi}_i(\mathbf{x}))^{-1} \mathbf{\Phi}_i(\mathbf{x})^H \cdot \mathbf{R}_i \quad (5)$$

with

$$\mathbf{R_i} = [R_i(0), R_i(1), ...R_i(K-1)]^T \quad (6)$$
$$\mathbf{\Phi}_i(\mathbf{x}) = [R_1(0), R_1(1)e^{\Delta\tau_i(\mathbf{x})w_1}, .., R_1(K-1)e^{\Delta\tau_i(\mathbf{x})w_{K-1}}]^T$$

and eliminating all terms that are independent of $\mathbf{x}$, the least squares solution is:

$$\hat{\mathbf{x}}_{LS} = \arg\max_{\mathbf{x}} \sum_{i=2}^{M} \frac{1}{||\mathbf{\Phi}_i(\mathbf{x})||^2} |\mathbf{R}_i^H \mathbf{\Phi}_i(\mathbf{x})|^2 \quad (7)$$

Next, we extend this model to the multipath propagation scenario. The received signals can then be modeled as:

$$r_i(n) = \sum_{p=1}^{P_i} \alpha_{i,p} s(n - t_0 - \tau_{i,p}) + \eta_i(n), \quad n = 0, 1, ...K-1 \quad (8)$$

wheras P_i is the number of received paths of sensor i, $\tau_{i,p}$ is the delay corresponding to path p of sensor i and η_i is the random noise of sensor i. The multipath parameters α and τ are assumed constant throughout the observation length K. The reference signal $R_1(k)$ is assumed to have one path $P_1 = 1$. This assumption can be held in a system where the reference sensor is chosen regularly based on this property, for example by choosing the signal with the narrowest correlation peak and with the least number of additional peaks. Expressing the signals in the frequency domain, the least squares solution can be obtained by minimizing the following function:

$$\bar{Q}(\mathbf{x}) = \sum_{i=2}^{M} \sum_{k=0}^{K-1} |R_i(k) - \beta_{i,1} R_1(k) e^{\Delta\tau_i(\mathbf{x}) w_k} - I(k)|^2 \quad (9)$$

with $\beta_{i,p} = \frac{\alpha_{i,p}}{\alpha_{1,1}}$ and $\Delta\tau_{i,p} = \tau_{i,p} - \tau_{1,1}$ and $I(k) = \sum_{p=2}^{P_i} \beta_{i,p} R_1(k) e^{\Delta\tau_{i,p} w_k}$ being the unknown interference resulting from multipath propagation. The information about the transmitter position lies only in the delay of the first arrival path following:

$$\Delta\tau_{i,1} = (\tau_{i,1}(\mathbf{x}) - t_0) - (\tau_{1,1}(\mathbf{x}) - t_0) \quad (10)$$
$$= \frac{\sqrt{(x_T - x_i)^2 + (y_T - y_i)^2} - \sqrt{(x_T - x_1)^2 + (y_T - y_1)^2}}{c}$$

III. Novel Algorithm

The proposed algorithm is based on equations (7) and (9). It consists of four main steps that will be described in detail later in this section:

1) If possible, estimate an initial position using the first identified peaks from cross-correlated signal pairs and applying the least squares positioning algorithm presented in [3].
2) Eliminate the interference term $I(k)$.
3) Calculate weights for sensors $2, ..., M$ depending on the outcome of step 2.
4) Use the interference-eliminated signals as well as the calculated weights to search for the weighted least squares solution according to (7). If step 1 was successful, the grid search area is reduced to a smaller area around the initially estimated position. If not, the complete grid area is used.

In the first step, an initial position is calculated. For the time delay estimation, the cross-correlations of the different received signals with the reference signal are calculated and the first identified peak above a threshold γ is interpolated and estimated as the TDoA. It is then fed to the method presented in [3]. The goal of this step is to reduce computational complexity if possible by limiting the grid search area. Alternatively, a low grid resolution can be chosen, resulting in higher quantization errors of the obtained position estimate. Due to matrix singularities resulting from large errors or from unfavorable geometries, the least squares algorithm in [3] sometimes fails to deliver an estimate.

In the second step, the cross-correlation between the reference sensor and all other sensors is calculated and peaks above a defined threshold γ are identified as received signal paths. Since we're assuming a single path at the reference sensor, the first arrived path corresponds to the position dependent path and later paths are identified as interference and are gradually subtracted from the signal until only one correlation peak above the threshold remains. For each identified path, the TDoA is estimated and the gain is calculated according to eq. (5). With the estimated delay and gain $\Delta\hat{\tau}, \hat{\beta}$, the path is subtracted as:

$$\bar{R}_i(k) = R_i(k) - \hat{\beta} R_1(k) e^{-\Delta\hat{\tau} w_k}. \tag{11}$$

Figures 1 and 2 show an example of a signal with three paths before and after step 2. Signal $\bar{R}_i$ represents the interference-free signal after step 2.

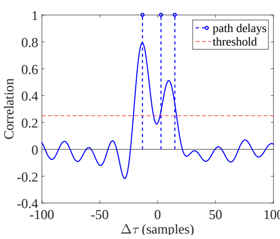

Fig. 1. Multipath propagated signal with 3 incoming path

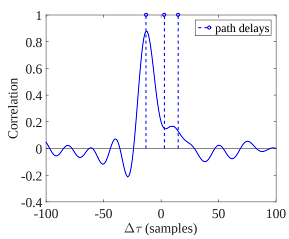

Fig. 2. Multipath propagated signal after undergoing step 1

If step 2 performs well, then cross-correlating the interference-eliminated signals with the reference signal would result in one high correlation peak. Therefor, the weights are calculated as the correlation coefficient of the highest peak after applying step 1. In the example in fig. 2, the weight would be 0.85.

The last step estimates the transmitter position by applying the algorithm:

$$\hat{\mathbf{x}}_{WLS} = \arg\max_{\mathbf{x}} \sum_{i=2}^{M} \frac{w_i}{||\mathbf{\Phi}_i||^2} |\bar{\mathbf{R}}_i^H \mathbf{\Phi}_i(\mathbf{x})|^2 \tag{12}$$

either on a large grid area or on a reduced grid area using the initial estimate from step 1.

The goal of the novel algorithm is to be able to apply the accurate one-step least squares algorithm to the complex scenario of multipath propagation, without undergoing a multidimensional search for the multipath parameters. With the second step of the algorithm being a rather simple step, the goal is to reduce the effect of remaining interference by applying the one-step solution instead of immediately introducing a bias to TDoAs through two-step solutions. This way, a simple yet robust algorithm can emerge as an answer for passive localization in multipath scenarios.

IV. Simulation Results

In this section, we show the performance of the position estimation using the presented algorithm. We compare results of the following algorithms:

- (i) 2S: A two-step least squares algorithm using the received signals (i.e., the initial estimate from step 1).
- (ii) PP-2S: A two-step algorithm using the pre-processed signals. After applying step 2 of the presented algorithm, the position is estimated by executing the same steps as in (i).
- (iii) PP-1S: The one-step weighted least squares algorithm using interference-free signals and, if possible, an initial estimate.

For the simulation, a geometrical setup of five sensors distributed on a circle with a radius of 700m was chosen. The position of the transmitter was chosen randomly for each simulation run within a $2000m \times 2000m$ plane. The generated transmit signal consisted of 300 symbols of band limited white Gaussian noise with a bandwidth of 1 MHz. The number of received paths per sensor as well as the parameters α and τ for each path were chosen from uniform distributions with $\alpha \in [0.05, 1]$ and $\tau \in [0.1, 20]$. The maximum number of paths to was set to p_{max}. For the minimum separation between delays, 0.1 of the symbol duration was chosen, allowing for overlapping paths scenarios to occur.

The simulation parameters shown in the results are:

- SNR: This is defined as the power of the received first path over the power of the received white Gaussian noise.
- SIR: This is defined as the power of the received first path over the power of the other received signal paths which are defined here as interference.
- p_{max}: The maximum number of paths to randomly chose from for each sensor.
- γ: The threshold, above which a correlation peak is identified as incoming signal path.

The performance criterion chosen for the results is the adjusted cumulative distribution function (CDF). It differs from the true CDF by not necessarily converging to 1. This happens whenever the algorithms fail to estimate a position. Additionally, a table is given for each plot with the failure rates of the algorithms.

A. Performance at different Signal to Interference Ratios

Fig. 3 shows the cumulative distribution function of the position estimation error for an SIR of 10 dB, 0 dB and -5 dB at an SNR of 10 dB. Table I shows the according failure rates of the two-steps algorithms due to large errors or bad geometries. The one-step algorithm, however, always results in a position estimate. At high SIR, the algorithms perform equally well. The lower the SIR, the bigger the advantage of the one-step algorithm, taking the failure rates into consideration. At equal power of signal and interference, the presented algorithm results in position estimates with 85% probability of an error below 200 m.

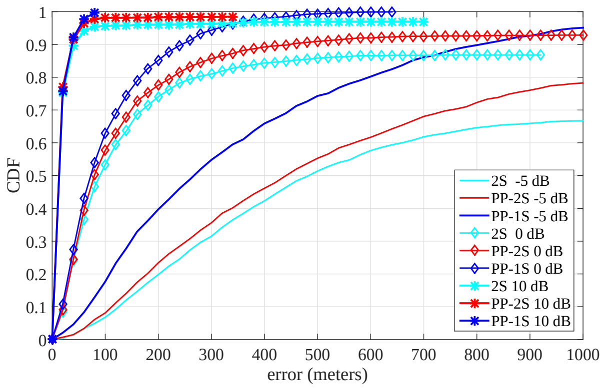

Fig. 3. Estimation error cumulative distribution function for $SNR = 10dB$ and $SIR = 10dB, SIR = 0dB, SIR = -5dB$. The maximum number of received paths is $p_{max} = 10$

	$SIR = 10dB$	$SIR = 0dB$	$SIR = -5dB$
2S	1.6%	15%	33%
PP-2S	0.95%	7.5%	8%

TABLE I
FAILURE RATES OF THE LOCALIZATION ALGORITHM AT DIFFERENT SIR VALUES

B. Performance at different Signal to Noise Ratios

Fig. 4 shows the cumulative distribution function for signal to noise ratios of 0 dB, 10 dB and 20 dB. Table II shows the failure rates of the two-steps algorithms. Again, the benefit of the new algorithm is higher for lower SNR. At SNR of 10 dB or 20 dB, 63% of the estimates obtained from the presented algorithm have an error below 300 m even for a signal to interference ratio of -3 dB. Even though the two-step algorithm shows better curves, it has very high failure rates of up to 38%. For an SNR of 0 dB, 50% of the estimates using the novel algorithm have an error below 300 m, while 50% of the estimates using the one-step algorithm have an error below 600 m, if we consider all results of the algorithm including the failures as 100%.

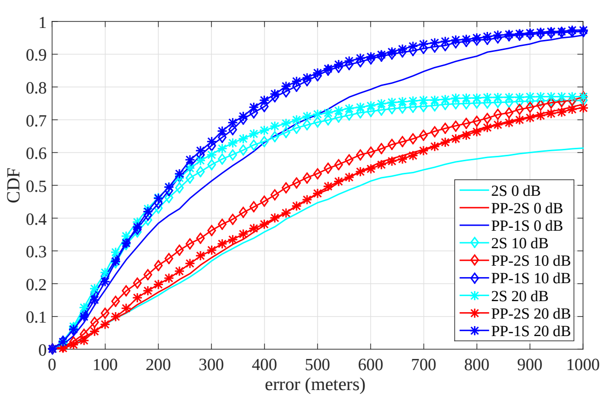

Fig. 4. Estimation error cumulative distribution function for $SIR = -3dB$ and $SNR = 0dB, SNR = 10dB, SNR = 20dB$. The maximum number of received paths is $p_{max} = 10$

	$SNR = 0dB$	$SNR = 10dB$	$SNR = 20dB$
2S	38%	24%	23%
PP-2S	8%	9%	9%

TABLE II
FAILURE RATES OF THE LOCALIZATION ALGORITHM AT DIFFERENT SNR VALUES

C. Performance at different Maximum Number of Paths

Fig. 5 shows the performance of the algorithms for different numbers of p_{max}. For higher p_{max}, the algorithms perform worse even at equal SIR. This is because the different paths are harder to resolve for a higher number of incoming paths. Again, the novel algorithm performs best considering the failure rates of the two other algorithms given in table III.

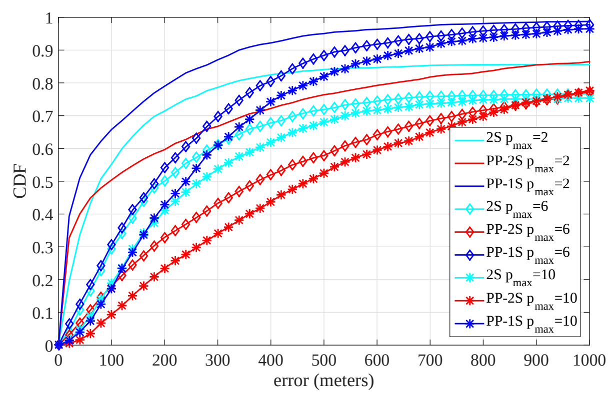

Fig. 5. Estimation error cumulative distribution function for SIR of -3 dB and an SNR of 10 dB over different p_{max}

	$p_{max} = 2$	$p_{max} = 6$	$p_{max} = 12$
2S	14%	23%	24%
PP-2S	6%	9%	9%

TABLE III
FAILURE RATES OF THE LOCALIZATION ALGORITHM AT DIFFERENT p_{max}

D. Performance at different Thresholds

Fig. 6 shows how the choice of the threshold affects the performance of the algorithms. Is γ chosen too small, then correlation peaks appearing due to noise will be identified as received signal paths. Is γ chosen too high, then some interference paths will remain unrecognized and will not be eliminated. The choice of γ depends on the SNR. For a wide range of SNRs, a threshold of 0.3 performed best.

The choice of the threshold affects all algorithms because, depending on γ, the first path is identified and estimated as the TDoA. That's why the failure rates are so high for $\gamma = 0.1$. The two-steps algorithms identify a wrong peak as the TDoA and the algorithm fails, whereas the one-step algorithm is far more robust against the wrong choice of γ. At $\gamma = 0.5$, much of the interference won't be neither identified nor eliminated. In this case, the one-step algorithm performs worse than for

$\gamma = 0.3$, which shows the importance of the interference-elimination step for this algorithm.

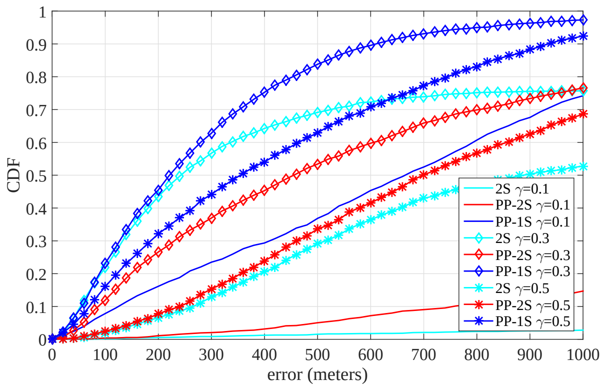

Fig. 6. Estimation error cumulative distribution function for $SIR = -3dB$ and three different thresholds. The maximum number of received paths is $p_{max} = 10$

	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$
2S	96%	24%	44%
PP-2S	70%	9%	10%

TABLE IV
FAILURE RATES OF THE LOCALIZATION ALGORITHM AT DIFFERENT THRESHOLDS

V. CONCLUSION

This paper presented a fully passive position estimation algorithm using distributed sensors in multipath scenarios. The algorithm doesn't assume knowledge of the number of received signal paths, the signal transmit time or the transmitted signal. The combination of the rather simple interference elimination and the mathematically reliable one-step least squares solution makes it robust against errors caused by multipath propagation. Remaining interference due to unresolvable multipath is not directly influencing the estimate by resulting in a TDoA bias. Simulation results confirmed that by showing how the algorithm is less sensitive to unresolvable or unrecognized remaining interference in the signals. Additionally, the algorithm presents a reliable approach due to its zero failure rate. All in all, the presented algorithm offers a solution which is robust against high noise power, high interference power, a high number of interference paths or a bad choice of the threshold.

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